Super-Exponential RE Bubble Model with Efficient Crashes

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Abstract

We propose a dynamic Rational Expectations (RE) bubble model of prices with the intention to exploit it for and evaluate it on optimal investment strategies. Our bubble model is defined as a geometric Brownian motion combined with separate crash (and rally) discrete jump distributions associated with positive (and negative) bubbles. We assume that crashes tend to efficiently bring back excess bubble prices close to a "normal" or fundamental value ("efficient crashes"). Then, the RE condition implies that the excess risk premium of the risky asset exposed to crashes is an increasing function of the amplitude of the expected crash, which itself grows with the bubble mispricing: hence, the larger the bubble price, the larger its subsequent growth rate. This positive feedback of price on return is the archetype of super-exponential price dynamics, which has been previously proposed as a general definition of bubbles. Our bubble model also allows for a sequence of small jumps or long-term corrections. We use the RE condition to estimate the real-time crash probability dynamically through an accelerating probability function depending on the increasing expected return. After showing how to estimate the model parameters, we examine the optimal investment problem in the context of the bubble model by obtaining an analytic expression for maximizing the expected log of wealth (Kelly criterion) for the risky asset and a risk-free asset. We also obtain a closed-form approximation for the optimal investment. We demonstrate, on seven historical crashes, the promising outperformance of the method compared to a 60/40 portfolio, the classic Kelly allocation, and the risky asset, and how it mitigates jumps, both positive and negative.

Keywords: financial bubbles, efficient crashes, positive feedback, rational expectation, Kelly criterion, optimal investment

JEL: C53, G01, G17

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