Bitcoin Bubble Trouble[1]

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Abstract

We present a dynamic Rational Expectations (RE) bubble model[5] of prices with the intention to evaluate it on optimal investment strategies applied to Bitcoin[6]. Our bubble model is defined as a geometric Brownian motion combined with separate crash (and rally) discrete jump distributions associated with positive (and negative) bubbles. The RE condition implies that the excess risk premium of the risky asset exposed to crashes is an increasing function of the amplitude of the expected crash, which itself grows with the bubble mispricing: hence, the larger the bubble price, the larger its subsequent growth rate. We use the RE condition to estimate the real-time crash probability dynamically through an accelerating probability function depending on the increasing expected return. We examine the optimal investment problem in the context of the bubble model by obtaining an analytic expression for maximizing the expected log of wealth (Kelly criterion) for the risky asset and a risk-free asset. Using our bubble model on Bitcoin from 8-Jul-2013 until 19-Dec-2017 would have generated a CAGR of 140% with a maximum drawdown of 69% giving a Calmar Ratio of 2.03. It would have moved out of Bitcoin gradually since 25-Apr-2017 to be completely out on 19-Dec-2017, three days before the crash. The outperformance of the Efficient Portfolio over just investing in Bitcoin was 265%, accomplished over 117 rebalances from 08-Jul-2013 to 20-Dec-2017. This strategy could thus afford a cost of 2.27% at each rebalancing period and still outperform investing only in Bitcoin.

Keywords: bitcoin, financial bubbles, efficient crashes, positive feedback, rational expectation, Kelly

JEL: C53, E47, G01, G17

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[1] A slightly different version is published in Wilmott Magazine.

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[4] Original runs for the results here were made on 20-Dec-2017. Summary of results were posted on LinkedIn on 22-Dec-2017 available at https://www.linkedin.com/pulse/yes-we-called-bitcoin-bubble-jerome-kreuser/ .

[5] This report is based on our paper: Kreuser, Jerome and Didier Sornette. 2017. "Super-Exponential RE Bubble Model with Efficient Crashes", Submitted for Publication in The European Journal of Finance and available on http://ssrn.com/abstract=3064668 . You will also find a version and additional information at http://riskontroller.com/info-center/ . Updated results after the crash will be posted there.

[6] Data supplied by Jan-Christian Gerlach (Doctoral Student in the chair of Entrepreneurial Risks at ETH Zürich) from Thomson Reuters Datastream.

Introduction

Everyone asks the question whether Bitcoin is in a bubble?

But that is the wrong question!

The question is; how do we exploit the information contained in the Bitcoin price?

We give our answer to this question using our bubble mitigation framework, which is detailed in (Kreuser and Sornette, 2017). The framework consists of a new rational expectations bubble model and an asset allocation method to test the framework based upon Kelly's criterion[7] (maximize the expected log of wealth) applied to our bubble model. We assume that asset prices are composed of Brownian motion plus jumps and frame our bubble model based upon that. We then show how to apply a Kelly method in that framework. In our 2017 paper, we show results for several historical bubbles including those developing in Hong Kong 1997, Brent Oil 2008, Dow Jones 1929, S&P 500 2007, Russia 1997, and Gold 2013-2017.

> ***Using our bubble model on Bitcoin from 8-Jul-2013 until 19-Dec-2017 would have generated a CAGR of 140% with a maximum drawdown of 69% giving a Calmar Ratio of 2.03. It would have moved out of Bitcoin gradually from 25-Apr-2017 until it moved totally out on 19-Dec-2017 with the rest invested in the risk-free rate. (Transaction costs are not included here but see below). On 19-Dec-2017 our bubble model estimated that Bitcoin would drop to 6,554 with a .78 probability. Although we did not give a time when that would occur, Bitcoin closed at 6,636 on 6-Apr-2018.***

Many people have stated that Bitcoin was in a bubble:

- Janet Yellen called it a "highly speculative asset". CNNMoney, 13-Dec-2017.
- Paul Krugman implies it is the most obvious bubble. Business Insider, 15-Dec-2017.
- Rich Ross says, "it is a classic bubble …". Financial Times, 30-Nov-2017.
- Ron Paul argues that cryptocurrencies are in an "exponential bubble". CNBC, 20-Dec-2017.
- And many others.

However, nobody says what to do about it except to avoid it.

We show in the following how our bubble model exploits the price information. We first summarize the bubble model, describe the asset allocation using Kelly's procedure, and then apply them to Bitcoin.

[7] MacLean et al., 2010.

The Bubble Model

We define the following set of variables:

Δt = discrete time interval $[t, t+1]$.

p_t = price of the risky asset at time *t*.

$\bar{r}_t$ = expected return of the risky asset on Δt when there is no crash or rally.

σ = standard deviation on Δt of the Brownian motion price process.

ε_t = sample from a standard normal distribution at time *t*.

r_D = discount rate of the asset price on Δt.

r_N = growth rate of the "normal price" on Δt.

r_f = risk-free rate on Δt.

p_0 = starting price of the risky asset.

$N_t = p_0 \exp(r_N t)$: this defines the normal price process.

ρ_t = probability that there is a correction (crash or rally) at time *t*.

$\kappa_i \in (-\infty, \infty)$ = the size of the i^{th} corrective jump relative to the distance to the normal price. We refer to it as the "crash factor".

η_i = probability (independent of ε_i) that, when there is a correction, it is of size κ_i.

$\bar{K} \equiv \sum_{i=1}^{n} \eta_i \kappa_i$ = expected corrective crash size relative to the distance to the normal price or expected crash factor.

$q_t = \frac{N_t}{p_t}$ is the relative (negative) mispricing of the risky asset.

We introduce the simple stochastic price process with a discrete Poisson process.

$$p_{t+1} = p_t \exp(\bar{a}_t + \sigma \varepsilon_t) \quad \text{with} \quad p_0 > 0$$

and

$$\bar{a}_t = \begin{cases} \bar{r}_t & \text{with probability } 1-\rho_t \quad \text{with } 0 \le \rho_t < 1 \\ \kappa_i \ln(q_t) + r_D & \text{with probability } \rho_t \eta_i \quad i = 1,2,\ldots,n \\ & \kappa_i \in \Omega \equiv \{\kappa_i \mid -\infty < \kappa_i < \infty,\ i = 1,2,\ldots,n\} \end{cases}$$

with (1)

$$q_t = \frac{N_t}{p_t} \quad \text{and} \quad \sum_{i=1}^{n} \eta_i = 1 \quad 0 < \eta_i < 1 \text{ and } \bar{K} = \sum_{i=1}^{n} \eta_i \kappa_i$$

$$N_t = p_0 \exp(r_N t)$$

When we apply our model to real data, we will want to assume that r_D and r_N vary over time. A positive κ_i with a $q_t < 1$ means the risky asset is in a positive bubble with a potential correction relative to N_t of size κ_i. A negative κ_i with $q_t > 1$ means that the risky asset is in a regime of transient under-valuation, where the price progressively accelerates downward and will eventually rebound in a rally jump of positive size κ_i times the mispricing amplitude to get closer to the normal price process. The price process model defined by (1) holds for positive ($\ln(q_t) < 0$) and negative ($\ln(q_t) > 0$) bubbles. In general, and in applications to actual price processes, we will assume that there is a separate distribution Π^+ for positive and Π^- for negative bubbles, consistent with empirical observations.

We assume that the crash probability is independent and constant over time: $E_{t-1}[\rho_t] = E[\rho_t] \equiv \bar{\rho}$: We will relax this assumption later and make it dynamic. At each time step, there is a probability $\bar{\rho}$ for a crash/rally to happen with an amplitude that is proportional to the bubble size, or amplitude defined as $\ln(q_t) = \ln\left(\frac{N_t}{p_t}\right)$ where $N_t = p_0 \exp(r_N t)$ and r_N is defined as the long-term average return[8].

We assume now that the expected return $\overline{r_t}$ is determined in accordance with the Rational Expectation[9] (RE) condition $E\left(\ln\left(\frac{p_{t+1}}{p_t}\right)\right) = r_D \quad \forall t$, which reads

$$\begin{aligned} E_t\left[\ln\left(\frac{p_{t+1}}{p_t}\right)\right] &= (1-\bar{\rho})\overline{r}_t + \bar{\rho}\left(\sum_{i=1}^{n}\eta_i\kappa_i\right)\ln(q_t) + \bar{\rho} r_D \\ &= (1-\bar{\rho})\overline{r}_t + \bar{\rho}\bar{K}\ln\left(\frac{N_t}{p_t}\right) + \bar{\rho} r_D \qquad (2) \\ &= r_D \end{aligned}$$

where $\bar{K}$ is the expected crash factor.

[8] We call it the "normal price return". Some may interpret this as a fundamental price return but that is not the specific intention here.
[9] See Blanchard and Watson, 1982.

With the RE equation, the value $\overline{r_t}$ of the expected return of the risky asset is:

$$\overline{r_t} = r_D - \frac{\overline{\rho}\overline{K}\ln(q_t)}{1-\overline{\rho}} \tag{3}$$

Equation (3) expresses a positive feedback of the price p_t on the return $\overline{r_t}$ that drives the price process before the crash occurs: the larger the price p_t above the fundamental or normal price N_t, the larger the expected return. As seen from equation (3), this positive feedback results from the assumption that the amplitude of the crash, when it occurs, is proportional to the mispricing $\ln(q_t) = \ln\left(\frac{N_t}{p_t}\right)$. Equation (3) not only relates positively the instantaneous $\overline{r_t}$ to the average crash probability $\overline{\rho}$ and to the average crash factor $\overline{K}$, but also to the log-price, $\ln(p_t)$, in excess to the logarithm of the normal price. This corresponds to an "efficient crash" condition, in the sense that the crash sizes are approximately proportional to the amplitude of the bubble so that the price recovers a value close to the normal price after a crash. The crash is an efficient correction to mispricing in that the price will oscillate about the normal price converging in the limit of long times.

Quantitatively, the model is mildly explosive, similarly to (Phillips and Yu, 2010 and Phillips, Wu and Yu, 2009), since the return in a positive bubble increases with only the logarithm of the price, and not as a positive power of the price as for instance in (Corsi and Sornette, 2014). We will introduce super-exponential exploding bubbles by allowing the probability of a crash to be a function of time and to depend upon the mispricing. In that case, we can also generate finite-time singularities (Sornette and Cauwels, 2015).

Exploiting the Bubble Model

Let λ_t be the fraction of wealth, W_t, allocated to the risky asset in time t and $1-\lambda_t$ the allocation to the risk-free asset with return r_f. Then

$$W_{t+1} = \left(\lambda_t \exp\left(\overline{a}_t + \sigma\varepsilon_t\right) + \left(1-\lambda_t\right)\exp\left(r_f\right)\right)W_t \tag{4}$$

where $\overline{a}_t$ has been defined in (1). We wish to determine $\max_{\lambda_t} E\left[\ln\left(\frac{W_{t+1}}{W_t}\right)\right] \equiv L\left(\lambda_t^*\right)$.

For simplicity, we use ρ in place of $\overline{\rho}$ or ρ_t in the following.

Define the shifted lognormal process $Y\left(s_{t,}\lambda_t\right) \equiv \exp\left(r_f\right) + \lambda_t\left(\exp\left(s_t + \sigma\varepsilon\right) - \exp\left(r_f\right)\right)$. Then we have

$$
\begin{aligned}
E\left(Y\left(s_t,\lambda_t\right)\right) &= \exp\left(r_f\right) + \lambda_t\left(\exp\left(s_t + \frac{\sigma^2}{2}\right) - \exp\left(r_f\right)\right) \\
Var\left(Y\left(s_t,\lambda_t\right)\right) &= \lambda_t^2\left[\exp\left(s_t + \frac{\sigma^2}{2}\right)\right]^2\left(\exp\left(\sigma^2\right) - 1\right)
\end{aligned}
\tag{5}
$$

We use the following expression (Elton and Gruber, 1974) valid for a log-normally distributed random variable Y :

$$
\begin{aligned}
E\left[\ln\left(Y\right)\right] &= \ln\left(E\left(Y\right)\right) - \frac{1}{2}\ln\left(1 + \frac{Var\left(Y\right)}{\left(E\left[Y\right]\right)^2}\right) \\
&= \ln\left(E\left(Y\right)\right) - \frac{1}{2}\ln\left(\left(E\left[Y\right]\right)^2 + Var\left(Y\right)\right) + \frac{1}{2}\ln\left(\left(E\left[Y\right]\right)^2\right)
\end{aligned}
\tag{6}
$$

Then the expression for the expected log of wealth (Kelly) follows as:

$$
L\left(\lambda_t\right) \equiv E\left[\ln\left(\frac{W_{t+1}}{W_t}\right)\right] = \left(1-\rho\right)E\left[\ln\left(Y\left(\overline{r}_t,\lambda_t\right)\right)\right] + \rho E\left[\ln\left(Y\left(\overline{\mathrm{K}}\ln\left(q_t\right) + r_D,\lambda_t\right)\right)\right] \tag{7}
$$

We substitute (56) in (67) and use that in (78) to get an explicit expression without he expectation. It is the function (78) that we wish to maximize.

It is a continuously differentiable function defined except at the boundaries where the log argument is zero. Therefore, we can apply nonlinear optimization methods where we do not need to supply explicit derivatives. Kreuser and Sornette (2017) estimate an explicit approximation for the optimal λ_t^* as:

$$\lambda_t^* \approx \frac{r_D - r_f + \frac{\sigma^2}{2}}{\sigma^2 + (1-\rho)\left(\overline{r} - r_f + \frac{\sigma^2}{2}\right)^2 + \rho\left(\overline{\mathrm{K}}\ln(q_t) + r_D - r_f + \frac{\sigma^2}{2}\right)^2} \tag{8}$$

We calculate estimates of the parameters $\Delta t, \sigma, \eta_i, \kappa_i, \rho_t$ by applying the methods of Jacquier and Okou (2014) and Audrino and Hu (2016) to separate the Brownian motion from jumps in historical data.

Then we estimate r_D, r_N on historical price data selecting a shorter window size for r_D and a longer one for r_N by fitting an exponential without anchoring. Optimal window sizes are selected.

We extend our analysis to real-time bubble evaluation by assuming that the probability varies with time. One way to do this is by reversing the RE equation to estimate the crash/rally probability as in equation (99)below. We can do this directly if we can estimate $\overline{r}_t$. But, as we know, estimating the expected return can be a "fool's errand"[10]. However, how big of a fool's errand depends on the relationship between the size of $\overline{r}_t$ and σ. Assuming that σ is bounded, it becomes ever easier to estimate $\overline{r}_t$ as it accelerates.

$$\overline{r}_t = r_D - \frac{\rho_t \overline{K}\ln(q_t)}{1-\rho_t} \quad \text{or} \quad \rho_t = \frac{\overline{r}_t - r_D}{\overline{r}_t - r_D - \overline{K}\ln(q_t)} \tag{9}$$

We have another alternative. We assume that the probability of a crash is a function of the mispricing, $\rho(q_t)$, and seek to estimate that functional form. We assume a parametric form for the probability for a positive bubble ($q < 1$) and for a negative bubble ($q > 1$) of the form:

$$\rho(q) \equiv \left\{ \frac{1-q^a}{1+b} \quad b > -1 \quad \begin{array}{l} \text{if } q < 1 \quad \Rightarrow a > 0 \\ \text{if } q > 1 \quad \Rightarrow a < 0 \end{array} \right. \tag{10}$$

For a positive or negative bubble, we have $0 < q^a \le 1$ and $a\ln(q) \le 0 \quad \forall a$ as given above.

If we have $-1 < b < 0$, we define $\rho(q) \equiv 1$ for $q^a \le -b$. The case $-1 < b < 0$ results in a finite-time singularity with super-exponential growth as in Johansen and Sornette (2010) and as confirmed in a model-independent analysis of real stock market data by Leiss *et al.* (2015) and in equity markets by Yan *et al.* (2012). This is because when the denominator in (10) is <1, the corresponding numerator can attain the

[10] Merton, 1980.

value of the denominator with finite mispricing and thus in finite time. For a value of $\rho = 1$, $\overline{r_t}$ becomes plus or minus infinity as shown in equation (99) assuming that $\bar{K}\ln(q_t) \neq 0$. However, that never occurs as the crash probability converges to one and the price crashes before $\overline{r_t}$ becomes infinite. We estimate a, b using $r_t \equiv \ln\left(\frac{p_{t+1}}{p_t}\right) = \overline{r_t} + \sigma\varepsilon_t$ and equation (99). It is this approach that we apply to the Bitcoin analysis using (8) to calculate λ_t^*.

Different variables in equation (99) can be estimated with different degrees of accuracy. It is this problem of data reconciliation that is worthwhile to investigate further in future work.

Application to Bitcoin

Our bubble model created all the following figures. Figure 1 contains five plots consisting of:

1. Optimal efficient portfolio value: consisting of an optimal allocation between the asset and the risk-free rate (using 2%), based upon the solution obtained from our bubble model using the Kelly criterion applied to our model that quantifies risks as a mixture of continuous volatility and jumps.
2. Actual Bitcoin price: The actual price of the risky asset measured in logs in the graphs.
3. Normal price: This is the price that Bitcoin is expected to move back towards.
4. BM Kelly portfolio: The classical Kelly allocation between Bitcoin and the risk-free rate that quantifies risks solely based on return volatility.
5. The 60/40 portfolio: 60% in the asset and 40% in the risk-free rate.

We make a few observations on Figure 1.

1. Allocating to Bitcoin using our bubble model from 8-Jul-2013 with a Kelly allocation would have outperformed all the other methods specified.
2. Our bubble model provided a compound annual growth rate (CAGR) of 140% with a maximum drawdown of 69% giving a Calmar Ratio of 2.03 and a Sharpe Ratio of 1.28.
3. We see that the Efficient Portfolio value is nearing the Bitcoin price, but it is still 2.65 times greater. It is flattening because it moves gradually out of Bitcoin from 25-Apr-2017 to 19-Dec-2017, anticipating a possible decline.
4. We see the Efficient Portfolio value is converging toward the Brownian Motion Kelly Portfolio value. However, that is because the BM Kelly Portfolio is accelerating as it is taking no

consideration of the possibility of a crash. As the BM Kelly Portfolio is fully leveraged, it will drop much further if the Bitcoin price crashes.

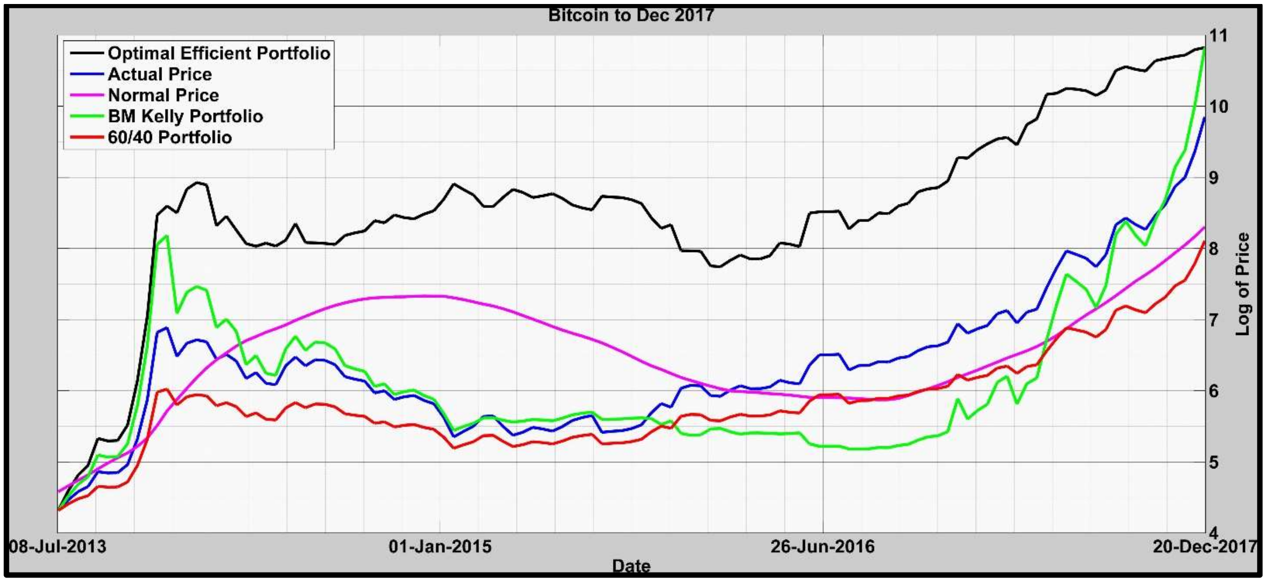

Figure 1: Bitcoin price, the normal price, and portfolio comparisons. The optimal Efficient Portfolio is a mixture of the asset (Bitcoin) and the risk-free rate. Lambda (λ) is allocated to Bitcoin and $(1-\lambda)$ to the risk-free rate.

We can see from the graph when Bitcoin entered a positive bubble and the exact date from the detailed analysis [11]. By our calculation based on the present model (1), Bitcoin entered and stayed in a positive bubble[12] phase around 16-Feb-2016.

Reading the values for $\overline{\mathrm{K}} = .68$, $q_t = .21$, and the probability $\rho = .78$ from Table 1, we have by our bubble estimates that the Bitcoin price could drop to $\$\,6,554 = 18,940\exp\left(.68\ln\left(.21\right)\right)$ with a probability of .78 following 19-Dec-2017. This should be compared to the value of $8,000 that Michael Novogratz has predicted[13]. However, his probability estimate of that value is unknown.

On 22-Dec-2017, bitcoin price dropped as low as $10,776 from an all-time high of $19,783.21 on Dec. 17, 2017.

[11] Not supplied here.
[12] A positive bubble is when the price exceeds the normal price and is expected to correct back towards the normal price at some time. One can see this in Figure 1.
[13] Michael Novogratz, Bloomberg Markets, December 22, 2017. He was planning to start Crypto Hedge Fund and has now shelved those plans. He states specifically that it may reach $8,000 but has no probability attached to that value.

The Figure 2 gives the values of λ overlaid with the Bitcoin price. In our present application, we restrict $\lambda \in [-1,2]$. If $\lambda = -1$, this means we short Bitcoin 100% and invest that into the risk-free rate[14]. If $\lambda = 2$, then we short the risk-free rate by 100% and invest that in Bitcoin. With a value near zero, it means invest everything in the risk-free rate. We see λ dropping near zero at the first crash. Then shorting Bitcoin through much of 2015 and then leveraging Bitcoin with some drops near anticipated crashes. Finally, it drops near zero from 25-Apr-2017 where it becomes closer to zero as we near 19-Dec-2017. We include here the values of λ and other values in the Table 1.

Importantly, on 19-Dec-2017, the λ value goes to zero when the Bitcoin price is at $18,940.57 while the value of the Efficient Portfolio over that period is **$50,206.47** compared to the Bitcoin price of **$18.940.57**.

Most importantly, on that date, we are not invested ($\lambda = 0$) in Bitcoin and experience no effect from its decline on 22-Dec-2017.

We did not include transaction costs in the calculation of the graphs in Figure 1. However, we note that the outperformance of the Efficient Portfolio over just investing in Bitcoin was 265.07%. This was accomplished over 117 rebalances from 08-Jul-2013 to 20-Dec-2017. This means that we could afford a rebalancing cost of 2.27% at each rebalancing period and still outperform investing only in Bitcoin.

[14] Implementing shorting efficiently might have been an issue, especially before the introduction by CBOE Global Markets Inc. of futures contracts on bitcoin in Dec. 10, 2017, and CME Group Inc launching of bitcoin futures on Sunday, Dec. 17.

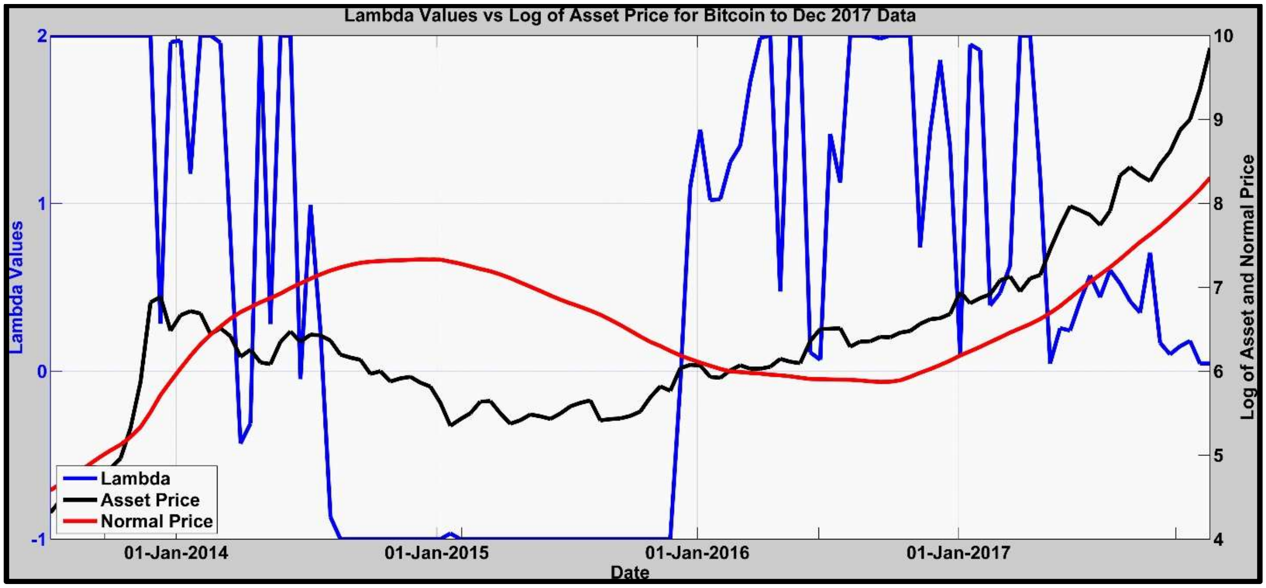

Figure 2: Values of λ overlaid with the log of the Bitcoin price.

We summarize the columns in the Table 1:

1. Crash size: the value K , which is the size of the crash back to the normal price (or up to the normal price in the case of a rally).
2. Crash Prob: The probability of a crash.
3. Rally size: Same as crash size for a rally.
4. Rally prob: The probability of a rally.
5. Qt – Mispricing: This is q_t, which is the value of the normal price divide by the actual price.
6. Lambda λ : This is the allocation to bitcoin.
7. Efficient Portfolio Value: This is the value of the portfolio applying our bubble model.
8. Price Bitcoin to USD: This is the Bitcoin price.

Table 1: Kelly portfolio performance using our efficient Crash-bubble model on Bitcoin 11-Apr-2017 to 19-Dec-2017

	Crash Size (Crash Factor)	Crash Prob	Rally Size	Rally Prob	qt Mis-pricing	Lambda λ	Efficient Portfolio Value	Price Bitcoin to USD
11-Apr-17	0.15	0.31	0.15	0.06	0.58	2.00	17,023.61	1,218.99
25-Apr-17	0.15	0.52	0.15	0.06	0.59	1.16	18,406.91	1,269.00
09-May-17	0.15	0.80	0.15	0.06	0.47	0.05	26,004.30	1,720.28
23-May-17	0.75	0.58	0.15	0.08	0.39	0.26	26,405.24	2,264.23
06-Jun-17	0.89	0.55	0.15	0.08	0.33	0.24	28,264.06	2,880.74
20-Jun-17	1.10	0.44	1.15	0.10	0.39	0.42	27,945.99	2,740.00
04-Jul-17	1.39	0.29	0.67	0.10	0.45	0.57	27,348.46	2,596.12
18-Jul-17	1.75	0.27	0.79	0.08	0.55	0.44	25,601.65	2,303.71
01-Aug-17	1.35	0.29	0.97	0.06	0.51	0.60	27,705.45	2,731.00
15-Aug-17	1.67	0.24	0.84	0.06	0.37	0.52	36,419.50	4,155.67
29-Aug-17	4.41	0.17	0.73	0.06	0.37	0.42	38,370.18	4,578.82
12-Sep-17	2.28	0.28	0.66	0.06	0.45	0.35	36,968.03	4,172.56
26-Sep-17	2.28	0.14	0.15	0.08	0.53	0.71	36,106.87	3,888.03
10-Oct-17	2.28	0.62	0.15	0.08	0.48	0.17	41,767.13	4,749.29
24-Oct-17	1.22	0.73	0.15	0.08	0.46	0.10	42,950.55	5,523.40
07-Nov-17	1.22	0.64	0.15	0.08	0.39	0.15	44,239.07	7,130.28
21-Nov-17	1.22	0.61	0.15	0.08	0.39	0.18	45,159.44	8,095.23
05-Dec-17	0.68	0.71	0.15	0.08	0.30	0.05	48,795.59	11,677.00
19-Dec-17	0.68	0.78	0.87	0.10	0.21	0.00	50,206.47	18,940.57

It may be argued that our bubble model was only successful because we started it so early, 8-Jul-2013. So, we started it from 5-Jan-2015. We did adjust the historical window size for the parameters r_D, r_N . The corresponding result is given in Figure 3. We see that again the Optimal Efficient Portfolio is leveling off and that the BM Kelly Portfolio is accelerating up by leveraging and, so it will fall harder.

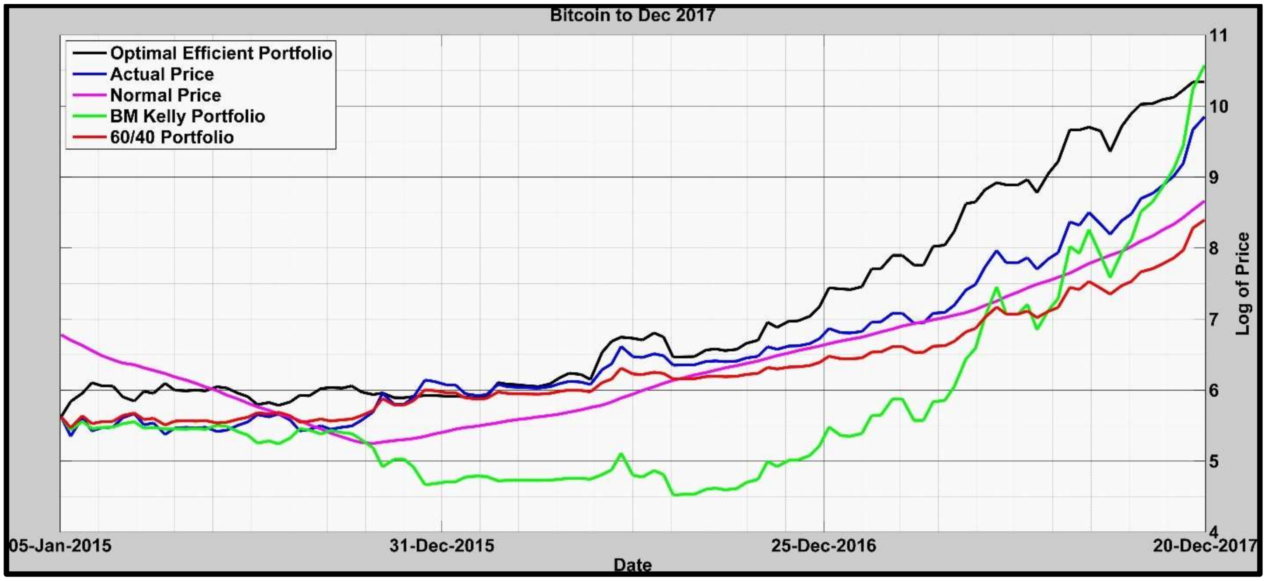

Figure 3: Bitcoin portfolio comparison beginning 5-Jan-2015.

Finally, we make a run from 1-Aug-2017 to demonstrate its behavior and the result is similar.

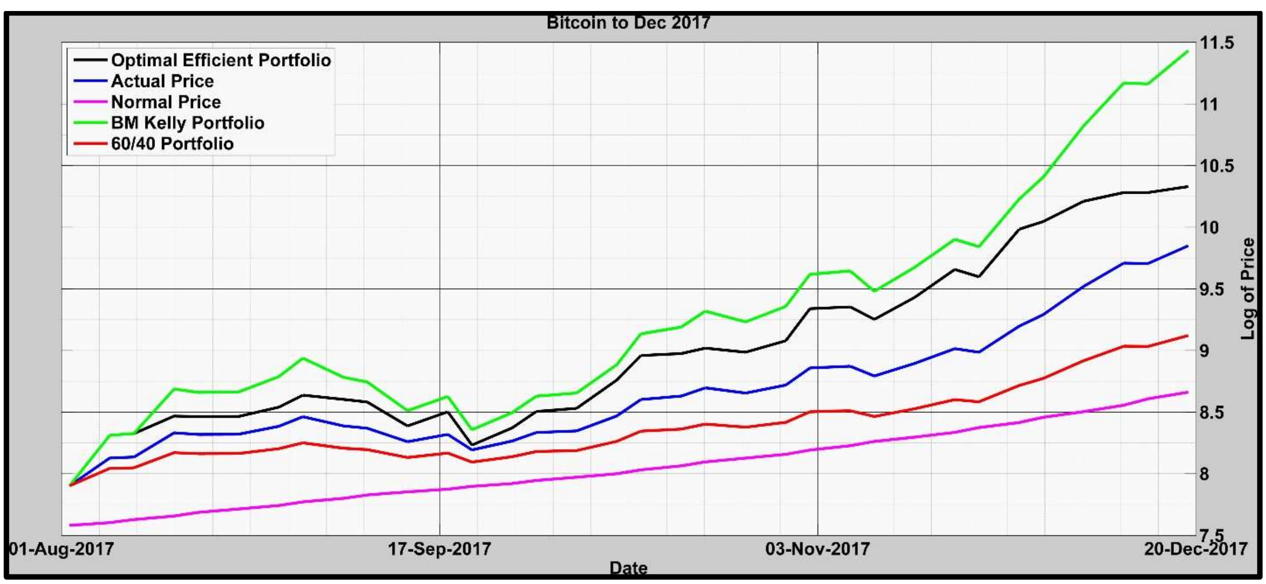

Figure 4: Bitcoin portfolio comparison beginning 1-Aug-2017.

Summary

We have proposed a rational expectations bubble model with "efficient crashes", with a positive feedback of over-pricing on crash probability, and have shown it to be consistent with many concepts usually invoked in classical bubble models. We have evaluated the bubble model by showing that, in combination with an optimal investment methodology, it can perform well on the bitcoin bubble that has developed and crashed in December 2017. Furthermore, it compared favorably to other portfolio methods such as the classical Kelly and a 60/40 portfolio as part of our evaluation.

Most importantly, on 19-Dec-2017 our bubble model estimated that Bitcoin would drop to 6,554 with a .78 probability. Although we did not give a time when that would occur, Bitcoin closed at 6,636 on 6-Apr-2018.

The crucial point here is not simply calling a bubble but rather the outperformance over time of hedging against crashes and advantaging rallies using our bubble mitigation technology.

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