A Sovereign Asset-Liability Framework with Multiple Risk Factors for External Reserves Management – Reserve Bank of India[1]

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1. Introduction

1.1. RBI and the financial crisis

The global financial crisis has impacted investment portfolios around the world, including those in many of emerging economies. The present Governor[5] of the Reserve Bank of India observed in early 2009 "… The crisis has called into question several fundamental assumptions and beliefs governing economic resilience and financial stability". The literature on systemic sudden stops emphasizes the importance of large current account deficits and foreign exchange denominated debt as common vulnerabilities in a large number of systemic sudden stops. India's current account deficit in Financial year 2009 was 3.1 percent of GDP, the same level as in 1991 when it had a massive balance of payments crisis. Liability dollarization in India has been limited due to strict controls and enforcement by RBI. In spite of the adverse impact, there are several factors that helped India to weather the crisis. First, the banking sector continued to function normally; Second, the high level of foreign exchange reserves provided confidence in the ability of RBI to manage any adverse balance of payments situations arising out of lower export demand and dampened capital flows; Third, headline inflation declined sharply in 2008-09, before it rose again in later 2009; Fourth, because of mandated agricultural lending and employment generating programs of the government, rural demand continued to be robust. Notwithstanding the above, the astonishing speed at which the government organized the fiscal stimulus (approximately 5.7% of GDP) and monetary stimulus (approximately 9% of GDP) ensured that the fall in output or even the collapse of output as happened in Argentina and is so typical of sudden stops was avoided. Calvo, 2009, emphasizes the role of "stock" issues in triggering crises as against "flows" which we factor into our model and whose results are consistent with the measures reported in Wyplosz, 2007.

At the same time, there are several recommendations for financial stability and measures to be followed for ensuring domestic as well as foreign currency liquidity and adoption of certain risk measures and controls. For example, there is a firm assurance to meet any demand-supply gaps of foreign exchange in the domestic foreign exchange market or in their operations to meet foreign exchange requirements of public sector oil marketing companies against oil bonds.

There was depletion in RBI's foreign exchange reserves for which valuation[6] losses accounted for about 60%. In addition, reserves have been spent as a response to foreign exchange adjustments during the financial crises. Furthermore, FDI dropped considerably over the crises period and this impacted the balance sheet of RBI. The ratio of net foreign assets to net domestic assets can also be seen as a signaling factor and this is important in managing the balance sheet.

Most importantly, it has been seen that a sufficient level of reserves has been of critical importance during the crises and that the priority should be to maintaining a stable growth in reserves levels and a floor on the level of reserves.

1.2. The reasons for a new framework for the Reserve Bank of India

Central bank foreign exchange reserves risk management concerns balancing many objectives and issues, ranging from broad macroeconomic policy objectives, such as monetary policy and foreign exchange management, to micro aspects, such as the definition of portfolio benchmarks and the evaluation of investment managers. Furthermore, constraints arising from legal, human resources, asset markets, and institutional set up affect the actual achievability and implementation of reserves management objectives. While the macroeconomic aspects of reserves management have been much analyzed, they continue to draw attention, especially the relationship between financial crises and reserves and external debt management. The micro aspects of improving reserves management have also received much more attention with the wider investment universe and the broader range of financial tools that have become available.

While these macro and micro issues have been considered for some time and are analyzed in much detail in various strands of literature, often these aspects are not being considered in an integrated manner, at least not within one common analytical and empirical framework. Furthermore, strategic risk management has lagged far behind because existing approaches tended to mimic those used by business firms, did not incorporate country-specific factors, lacked strategic interactions, often excluded trade flows and fiscal dimensions, had inadequate modeling flexibility, failed to dynamically realign portfolios, and the treatment of uncertainty was far from robust.

Our approach addresses these shortcomings and is based on a general, dynamic stochastic optimization model with a tree-based uncertainty structure. The numerical approach allows us to move beyond the classical methods of risk and return, efficient frontiers, and utility functions by considering much more general objective functions, behavioral relationships, equalities, and inequalities. The approach also innovates by allowing the user to view the full density functions of any outcomes that depend upon the optimal decisions taken. The user can then reshape these density functions as much as possible to desired profiles and obtain the resultant decisions to be taken today to achieve these profiles. As we will show, this approach allows senior management to define relatively broad objectives and constraints, and have those translated into an analytically rigorous approach, without giving up basic intuition and understanding.

This approach is discussed in detail in Claessens and Kreuser, 2007. Claessens and Kreuser, 2010 compares this approach to alternative methodologies in more detail. The initial framework for our approach was developed at the World Bank for sovereigns. It has also been applied to sovereign wealth funds in Claessens and Kreuser, 2010. Variations of it have been used in the reinsurance industry to handle catastrophic risks and the framework has been extensively published[7]. Furthermore, it is tailored to each specific institution or country separately along with each specific situation and this makes for a very powerful framework.

Our approach here has several important attributes. It is strategic and takes into consideration the long-term strategic objectives, polices, and multiple risk constraints. The processes or the uncertainty structure can change over time depending on the level and time of any of the other processes thus allowing extremes to be triggered by events, allowing changing volatility and correlations and therefore incorporating the concept of herding, allowing volatility clustering and mean or trend reverting, and therefore allowing for the incorporation of processes that change with respect to changing economic and financial regimes. The uncertainty structure therefore captures processes that have other than normal distributions. The approach allows a multitude of differing kinds of objectives including indicators, such as the IMF financial stability indicators or indicators that are being developed to measure the possibilities of financial instability and incorporating these into the asset allocation process. The approach can take on board all types of assets, liabilities, and derivatives and maintains a balance sheet at each point in time and at each event in time. The uncertainty structure is dynamic and allows for dynamic rebalancing that is an approach that is essential over a longer period of time especially when you have transaction costs, liquidity constraints, changing economic regimes, uncertain future cash flows extreme events, derivatives, and future binding constraints. In addition, the approach for generating the uncertainty structure does not just depend on history but on multiple factors including expert views, selected economic and financial theories, and implied prices.

The typical approach to mean-variance tends to have problems with stability of solutions with respect to the data. It is usually formulated as a single period model with no dynamic rebalancing. The distributions are usually assumed to be normal and there is difficulty in incorporating extremes. The use of variance as a measure of risk penalizes the upside and semi-variance is often used instead. However, this also does not capture extreme events and it is not a coherent risk measure, which simply means that it is not sub-additive and does not work well when combining measures of risk when portions of the portfolio are outsourced to several different institutions as is often the case for central banks. The use of VaR or parametric VaR does not lend itself to optimization, does not capture extremes, is not a coherent risk measure, and is not useful in strategic risk management.

With the general Monte Carlo simulation, one is unable to impose constraints and objectives and, therefore, it is less useful for asset allocation decisions. It can be useful to evaluate a portfolio in the shorter term but in the longer term strategic analysis it makes no sense not to allow the model to rebalance as factors change. Monte Carlo simulation can only allow for rebalancing in a procedure called the “Fixed-Mix” problem where the asset allocation remains fixed in the same proportions at all rebalancing periods – an assumption which is not sensible in the longer term. It has been show to be inferior to dynamic stochastic optimization, Ziemba and Ziemba, 2007, and cannot handle many of the issues discussed in the previous paragraphs. This matter, along with the contingent claims approach, Gray, 2007, are discussed in more detail in Claessens and Kreuser, 2007 and 2010.

As the above-mentioned issues have been discussed in the papers of Claessens and Kreuser listed in the references, we will not add more to these subjects now. What we want to emphasize in this paper is an approach to multi-dimensional risk factors. Normally, central banks are concerned with safety, liquidity, returns, and stability of various factors. In the case of the Reserve Bank of India there are seven factors that we consider to be important and show how to incorporate these over three rebalancing periods giving a total of 21 risk factors in our model.

2. RBI background facts on management of foreign exchange reserves

India’s foreign exchange reserves increased significantly in the first decade of the millennium, reaching an all-time high level of US$ 316.171 billion as of May 30, 2008. The following Table 1 shows the breakup of the reserves amongst its four components as of May 30, 2008 and on two more recent dates for a perspective on the movement in the size of reserves in recent years.

The fall in the size of reserves (FCA +gold) in 2008-09 was on account of intervention sales and valuation losses. Over the whole of 2008-09, net sale of US dollars was US$ 34.9 billion and valuation loss was US$ 37.7 billion. Consequently upon the purchase of 200 tones of gold from the IMF in October, 2009, the share of gold has gone up. RBI follows a policy of passive management of its gold holding.

Table 1: External Reserve in US$ million

Item	May 30, 2008	Feb 20, 2009	Feb 19, 2010
(a) Foreign Currency Assets (FCA)	306,203	239,821	254,203
(b) Gold	9,427	8,884	18,056
(c) Special Drawing Rights (SDR)	11	1	5,027
(d) IMF Reserve Tranche Position (RTP)	530	821	1,386
Total (a+b+c+d)	316171	249,527	278,672

Source: *Weekly Statistical Supplements – RBI*
Note: SDR and RTP are not on the books of RBI. From a reserve management perspective, only FCA and gold are relevant.

The Reserve Bank of India Act, 1934 (RBI Act) provides the over-arching legal framework for deployment of foreign exchange reserves.

RBI invests the FCA in multi-currency, multi-market fixed income assets. Going by the magnitude of valuation changes affecting the size of the FCA in US dollar terms, it is safe to conclude that RBI has maintained a highly diversified portfolio, comprising US dollar and a few other major convertible currencies. The universe for fixed income assets include deposits with other central banks, the Bank for International Settlements (BIS), top-rated foreign commercial banks, securities representing debt of top-rated sovereigns and supra national institutions with residual maturity not exceeding ten years and any other instrument or institutions as approved by its board of directors in accordance with the provisions of the RBI Act. Certain amendments to the law were carried out in 2006, which provided more flexibility in the use of derivative instruments, among others.

In recent years, FCA has been partitioned into several portfolios for reasons similar to those seen elsewhere viz. differentiated approach and focus in respect of the management of the liquidity portfolio, on the one hand, and longer-term investment portfolios, on the other. In RBI's own words, partitioning affords ‘better balancing of the various objectives of liquidity and return’.

However, by all indications, RBI takes a lot more currency risk compared to interest rate risk and credit risk. As will be explained soon, RBI significantly reduced its credit risk exposure in the wake of the global financial crisis. The broad pattern of investment of foreign currency assets (FCA) over the last six years is given in the following Table 2.

Two observations are important: (i) In 2007-08, the share of deposits with foreign commercial banks was brought down drastically and in an indiscriminate fashion, most possibly as a precautionary step with the emergence of the sub-prime turmoil and the share of deposits with other central banks and the BIS position was raised; (ii) In 2008-09, while the share of deposits with commercial banks remained at the same level as in the previous year, the share of securities was raised to an historically high level, while the exposure to other central banks and BIS was significantly curtailed. Incidentally, although RBI does not disclose its external asset managers’ program, it is likely that the aggregate amount involved is still modest. RBI discloses neither the currency composition of FCA nor the investment and performance benchmarks. However, we make use of the disclosure as per the SDDS template of the IMF.

Table 2: Investment pattern of foreign currency assets (FCA) in US$ million

As of March 31	FCA (a+b+c)	Securities (a)	Deposits with other central banks, BIS and IMF	Deposits with foreign commercial banks/Funds placed with external asset managers
2004	107,448	35,024 (32.6)	45,877 (42.7)	26,547 (24.7)
2005	135,571	36,819 (27.2)	65,127 (48.0)	33,625 (24.8)
2006	145,108	35,172 (24.3)	65,399 (45.0)	44,537 (30.7)
2007	191,924	52,996 (27.6)	92,175 (48.0)	46,753 (24.4)
2008	299,230	103,569 (34.6)	189,645 (63.4)	6,016 (2.0)
2009	241,426	134,792 (55.8)	101,906 (42.2)	4,728 (2.0)

(Figures in parentheses indicate proportion of FCA)
Source: *RBI, Annual Report 2008-09*

While safety and liquidity constitute the twin objectives of reserve management in India, return optimization becomes an embedded strategy within this framework. In terms of risk management, the Reserve Bank of India considers credit risk, currency risk, interest rate risk, liquidity risk, and operational risk. How to integrate most of these into a strategic framework is discussed in this paper.

Historically, the strategy of reserves accumulation was expected to continue due to global uncertainties. It has been suggested (Sivakumar, 2003) that while central bankers have not focused on profitable deployment of reserves that they also did not find a suitable measure for opportunity costs. We suggest a way to do that here. See also Gupta, 2008.

The reasons for holding reserves are to promote stability in the foreign exchange market, sustain confidence in the country’s ability to meet its external obligations, back the domestic currency of external assets, enable the government to meet its foreign exchange needs, and finally act as a buffer for dealing with national emergencies. By all indications, RBI has done a reasonably good job in meeting these objectives in recent years. We will also lay out a framework to handle most of these.

Adequacy of reserves has emerged as an important parameter in gauging RBI’s ability to deal with external shocks. With the changing profile of capital flows, the traditional approach of assessing reserve adequacy in terms of import cover has been broadened to include a number of parameters which take into account the size, composition and risk profiles of various types of capital flows as well as the types of external shocks to which the economy is vulnerable. In the early 1990s, as India emerged from its balance of payment crisis of 1990-91, a high level committee set up by the government under the chairmanship of Dr. C. Rangarajan, then a member of the country’s Planning Commission, recommended, among other things, that while determining the adequacy of reserves, due attention should be paid to payment obligations, in addition to the traditional measure of import cover of 3 to 4 months. In 1997, the committee that was set up for the purpose of preparing a road map for capital account convertibility suggested alternative measures of adequacy of reserves which, in addition to trade-based indicators also included money-based and debt-based indicators. Similar views were expressed by the second capital account convertibility committee in 2006.

In the recent period, assessment of reserve adequacy has been influenced by the introduction of new measures. One such measure requires that the usable foreign exchange reserves should exceed scheduled amortization of foreign currency debts (assuming no rollovers) during the following year – the so-called Greenspan-Guidotti rule and which is a measure we introduce in our framework.

The other one is based on a 'Liquidity at Risk (LaR)' rule that takes into account the foreseeable risks that a country can face. This approach requires that a country's foreign exchange liquidity requirement can be calculated under a range of possible outcomes for relevant financial variables, such as, exchange rates, commodity prices, credit spreads, etc. Reserve Bank of India has been undertaking exercises based on intuition and risk models to estimate LaR of the reserves. The traditional trade-based indicator of reserve adequacy, viz. import cover of reserves, which fell to a low of 3 weeks of imports at end-December 1990, rose to 11.5 months of imports at end-March 2002. The import cover continued to remain at this elevated level in the subsequent years as well and as at end-September, 2009 the import cover was 12.4 months. The ratio of short-term debt to foreign exchange reserves declined from 146.5 per cent at end-March 1991 to 12.5 per cent as of the end-March 2005. In the subsequent years, the ratio went up somewhat and as at end-September 2009 it was 15.1 per cent. The ratio of volatile capital flows (defined to include cumulative portfolio inflows and short-term debt) to the reserves declined from 146.6 per cent as at end-March 1991 to 35.0 per cent at end-March 2004. This ratio increased to 48.9 per cent as at end-September 2009 from 47.9 per cent in March 2009. We will also capture these issues in the model.

In the years preceding the global financial crisis 2007-09, there was almost a deluge of views and opinions suggesting that the reserves had exceeded the optimal level needed for intervention and financial stability purposes. Hence, by holding excess reserves, the country was foregoing significant investment and growth opportunities. As in certain other countries experiencing a rapid rise in reserves at that time, policy-makers and central-planners recommended the use of reserves for investment in priority areas, such as infrastructure. The experience of the global financial crisis has, however, meant a paradigm shift in the approach toward adequacy of reserves and currently it is almost universally acknowledged that the country needs to maintain a much higher level of reserves, not just for intervention purposes, as was thought hitherto, but also for providing foreign currency liquidity to domestic financial institutions at times of crisis. Wyploz, 2007, indicates that although reserves have risen considerably in nominal terms or as a percentage of GDP or exports, when reserves are linked to external liabilities, barring a few exceptions, there is no evidence of a massive reserve buildup. Central banks worldwide have probably responded to financial globalization which has certainly increased the risks by increasing their reserve holdings. Some of the Asian countries that have accumulated reserves, seemingly beyond the levels required, have done so to handle potentially large domestic financial threats especially in the banking system. We capture these concerns in the model also.

3. What can the RBI do now?

As has been seen, there are several issues that should affect the foreign currency allocation in the Reserves bank of India and in any other central bank. We are focusing here on those that we think are the most relevant to the RBI. So the issue becomes how to translate all these concerns into a framework suitable for analysis at the strategic level.

The RBI needs to adopt a robust framework and a model for strategy formulation that captures all the relevant concerns of the RBI. In order to do that, the RBI needs to enunciate its long-term goals, risks, policy constraint, and other important issues. This should be done outside of any consideration of any particular modeling framework but done in a general way. The next step is to adapt the modeling framework to the strategic issues. Our framework has the flexibility to make this happen.

Some of the relevant concerns are that the rapid globalization of the Indian economy requires reasonable protection against exogenous shocks and capital flows. There needs to be a better balance between currency, interest rate, and market risks and new asset classes have to be considered in this framework. The framework proposed here, dynamic stochastic optimization, using multiple CVaR constraints offers the flexibility to produce robust solutions that can provide stability in times of stress.

In order to do that, we combine the issues discussed in the preceding sections into a class of seven risk factors, which are:

- Lower limit on the size of reserves – US$ 200 billion, adjusted for nominal GDP growth (%)
- Lower limit on the ratio of NFA to (NFA+NDA)
- Upper limit on the (%) fall in value of reserves in any period in US dollars.
- Mark-to-market value of reserves not to lag behind the expected value, as measured in the composite currency
- Limit the liquidity at risk
- Foreign currency assets should exceed the amortization of external debt over the next 12 months
- Ratio of short-term external debt to reserves should not to exceed a pre-set level

These must all be formulated as risk factors because it is unlikely we can incorporate all of them together with fixed limits and obtain a feasible solution, especially since we generate values that are more extreme. Therefore, for example, we may require that with a 90% confidence level that the shortfall in the short-term debt to reserves ratio should not exceed 10% of the expected ratio. That risk constraint captures expectations on the tail but at the same time allows extreme values to occur as is natural. We discuss each of the seven risk constraints in more detail.

Lower limit on the size of reserves – US$ 200 billion, adjusted for nominal GDP growth (%): This is a very rough and ready number showing the bare minimum that is needed to ward off any sudden and cascading fall in market sentiment consequent upon an unexpected and adverse political, economic or security event acutely impacting the country. An indication about the order of the magnitude involved can be had by adding aggregate FII investment in equity and debt in India (currently around US$ 110 billion without marking to market. Market value will be much higher) and short-term external debt requiring redemption/roll over in the next 12 months. As of March 31, 2009 this number was US$ 93.3 billion.

Lower limit on the ratio of NFA to (NFA+NDA): This is a frequently-used criterion to signify the strength of the central bank's balance sheet in lending credibility and stability to the external value of the domestic currency.

Upper limit on the (%) fall in value of reserves in any period in US dollars: This is to ensure that a proportionate fall in the value of reserves, as measured in US dollars, in any period – say, over the next 12 months – arising out of exchange rate changes is not excessive. This constraint is an additional limit to currency risk. Currency risk is also handled in the overall structure of the model as all currencies are linked in the balance sheet. As will be argued in the following paragraph, the purchasing power of the reserves needs to be gauged in terms of an appropriate composite currency unit, but the US$ value of the reserves is critical from a market stability point of view. Like in most other countries, the domestic foreign exchange market is a market for US dollars and the most important exchange rate is the US$/INR rate.

Mark-to-market value of reserves not to lag behind the expected value, as measured in the composite currency: In the hierarchy of objectives for holding reserves by RBI, the precautionary motive, embodying the need for using the reserves for financing the country's imports and external debt services payments in the event of a severe BoP crisis, ranks very high. Seen from this perspective, the purchasing power of the reserves would be best measured in terms of a composite currency unit, reflecting the currency composition of imports and external debt. This constraint has been added to protect the value of reserves, expressed in the composite currency unit, from excessive downside risk on account of market volatility.

Limit the 'liquidity at risk': This constraint has assumed in the light of the global financial crises and a fairly tight constraint on the projected liquidity requirements is maintained. There are several reasons for the importance of this constraint. It is desirable to maintain a large pool of liquid assets for providing foreign currency liquidity in the event of a 'Sudden Stop'. India faced a sudden and sharp drop in certain key capital inflows in the wake of the collapse of Lehman Brothers in September, 2008 resulting in a shortage of foreign exchange funds amongst banks and non-bank corporates with a potential for disruption in the credit and foreign exchange market. This constraint helps prevent destabilizing expectations in the domestic foreign exchange market from taking hold in the event of any exogenous shock and it helps preserve the external liquidity and self-insurance properties of reserves. Furthermore, it helps to limit excessive risk-taking and in providing flexibility for portfolio rebalancing.

Foreign currency assets should exceed the scheduled amortization of external debt over the next 12 months: This is Greenspan-Guidotti rule.

Ratio of short-term external debt to reserves should not exceed a pre-set level: This constraint, though similar to Greenspan-Guidotti rule, is more relevant from the point of view of market sentiment, following the lessons of Asian crisis (1997-98). As mentioned in the RBI Annual Report 2008-09, the total short-term external debt, by residual maturity, was US$ 93.3 billion as of March 31, 2009, including convertible non-resident deposits at the domestic banks at US$ 32.1 billion. Although in normal times, non-resident deposits are most likely to be rolled over, in crisis situations non-residents could withdraw deposits in bulk – as was the case in 1990-91.

4. Analytical Framework for the RBI

The overall framework for this paper and for similar frameworks for other central banks in general can be found in Claessens and Kreuser, 2007. In this paper we discuss the framework in light of multiple risk constraints most relevant to the Reserve Bank of India. This goes beyond safety, liquidity, and returns that are the usual concerns of central banks. We incorporate all those risks discussed in the last section.

The size of this paper is limited by necessity. Therefore we discuss more details on the data generation, the model and the results in the paper Bhattacharya et al, 2010.

The approach is to estimate stochastic processes over changing regimes by stochastic differential equations determined by a combination of historical data, expert opinion, economic and financial theories, and implied prices where available. We begin the model projections on January 16, 2009 and define rebalancing times on January 16, 2010, 2012, and 2015, thereby giving a good set of time intervals for strategic planning. It is anticipated that the model will be rerun approximately every three months, when major changes are detected in the financial or economic regimes, or at any time for the purpose of stress testing.

The stochastic differential equations are used to define branching trees of various sizes or complexity. Then a dynamic stochastic optimization model is defined on the tree. Its description is independent of the typology of the tree and is easy to define as it is done in a form that looks like equations and is easy to read by a human or a computer, as discussed in Kreuser, 2004.

The risk constraints are defined as CVaR constraints, Rockafeller and Uryasev, 2002, so that they take into account the expectation on the tail of the distribution. As there are multiple constraints (21 in this case), the process of defining the values is one of initially selecting appropriate confidence levels and percentage losses on the shortfall and then refining them until an appropriate solution is obtained. It is also possible to minimize the sum of the CVaR constraints but this is useful in terms of getting an initial solution as the constraints can be on different measures; for example the ratio of debt to reserves versus the downside risk in the composite currency. Multiple risks are considered simultaneously, and the process is very efficient in practice.

Multiple shortfall values as risks are often formulated in the objective function; Zenios and Ziemba, 2007 give several examples of dynamic stochastic optimization models. The problem with this approach, however, is that when there are several different shortfall functions which often refer to very different factors; for example, ratio of debt to reserves and the shortfall in the composite currency. Therefore, they are difficult to combine into the objective. They also need to be formulated as convex functions to incrementally penalize the shortfall as it becomes larger and that convex shape is subjectively defined. Furthermore, that can make an otherwise linear problem nonlinear or piecewise linear. In both cases, the problem becomes more difficult to solve. This is so because it can limit the size of the uncertainty tree that can be computationally handled. Lastly, adding these shortfalls into the objective distorts the use of the dual or marginal information that we can make very good use of. The problem with multiple CVaR constraints, on the other hand, is that the initial values chosen may make the model infeasible. We find that this problem is not insurmountable and that there are ways to handle it.

Various measures of marginal returns, marginal contributions to each risk constraint, tail risk or CVaR marginal returns to confidence levels or shortfalls, opportunity costs, and state-price vectors are analyzed to determine asset rankings, opportunity costs of levels of reserves, importance of risk constraints, pricing of extreme events, interactions among states with respect to assets and risks, and other factors. These are mostly discussed in Bhattacharya et al, 2010.

5. Uncertainty Structure

The uncertainty structure is a tree that is build from the estimation of the parameters of the following stochastic process.

$$\frac{ds_i(t)}{s_i(t)} = \mu_i(s,t)dt + \sum_j b_{ij}(s,t)\sigma_j(s,t)d\omega_j(t) \qquad (1)$$

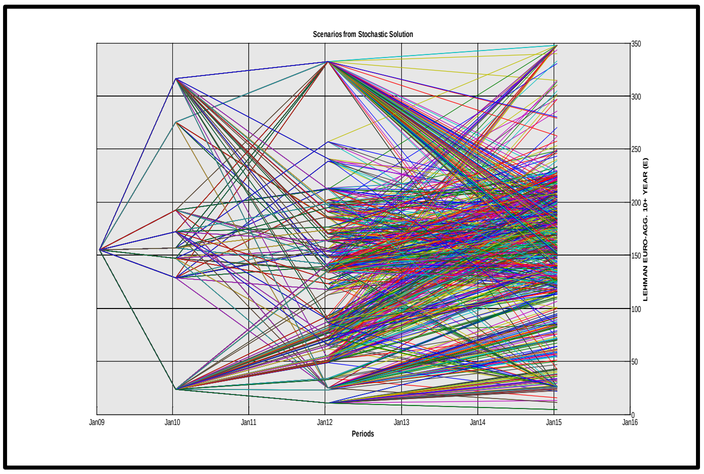
The parameters μ, b and σ are estimated from historical data, expert opinion, theories, and implied prices separately over the short term and over the long term. The extra degrees of freedom in estimating these parameter values allows for combining several factors together other than just the history of the process. The theories used for the price indices consist of mean- or trend-reversion. This is discussed in more detail in Claessens and Kreuser, 2007. All the factors are estimated together but they need not be. They can also change depending on time and level. The Econophysicist Joseph McCauley, 2004, considers (1) the “best tractable approximation to market dynamics”.

We estimate 27 processes using (1) over three time periods. The processes consist of Australia money market fund; Cash, short bonds, long bond, and corporate bonds in Euro, Sterling, USD, Yen; Central bank and BIS deposits; Exchange rates for all the currencies; India external debt by residual maturity; India total External debt; India net domestic assets; Liquidity requirements; and India GDP.

A binary tree of this many interdependent processes will not be feasible to solve[8]. The scenario tree needs to be built efficiently; small enough to solve yet big enough to capture the stochastics with a good collection of fat tail scenarios. This can be done using variable probability branches meaning that extremes can be added with fewer branches. The stochastic processes can be generalized to capture autocorrelation, but we do not do that here.

An example of one branching tree for one of the process is given in the following Figure 1. The mean-reversion is clearly visible here. It must be noted that the branches are not of equal probability.

Figure 1: Branching Tree



The dynamics of the branching tree are essential in the analysis of the problem. Dynamics matter whenever economic regimes undergo change or liquidity becomes an issue, transaction costs rise, future cash flows become unceratin, rare and special events occur, portfolios contain derivatives, binding (or infeasible) constraints come into play in future, and changing stochastics – all of these can happen in reality.

A good tree is one that is of solvable size, allows changing regimes, allows changes in volatility and correlations, estimates means that exhibit mean reversion and trending, estimates scenarios in fat tails and extreme events, reduces or eliminate arbitrage, allows stochastics as a function of time and level, allows for expert views, and provides good coverage of possible scenarios.

We judge the suitability of a tree by the quality of the decisions it gives us (not necessarily whether or not it matches distributions, Kaut and Wallace, 2003 and in Zenios and Ziemba, 2007. There are two ways to test suitability: *Stability* - meaning the same optimal value results from solving with different trees with same method; and *Bias* - does not introduce bias vis-à-vis the “true” solution. This second criteria cannot usually be tested in practice so we generally have to rely on our previous discussion of what makes a good tree and on stability. The framework proposed here works very well in both of these cases[9].

6. Model Structure

The importance of our framework is that it is very flexible and easy to tailor to each individual institution - in the stochastic process estimation, in the tree generation, in the model construction, and in the solution and analysis.

We must emphasize that in this paper the selection of the stochastic processes, the model structure, the risk constraints, and the portfolio composition are only estimated from information and data in respect of Reserve Bank of India ***that are in the public domain***. The purpose of this paper is to outline the process using this framework and we make no claim to any recommendations based upon the model results.

The equations of the model are outlined in the Annex. The model consists of a balance sheet for each currency separately with currency transfers allowed, incorporating transaction costs. The market prices of the assets are obtained from the stochastic processes on the tree and assets can be held or sold in whole or in part in any future rebalancing period on any event. The sale or purchase will incur transaction costs. The structure of the model, incorporating transaction costs tends to make it very stable, in general, and not subjected to the instabilities on the data that one usually finds in a one-period mean-variance model.

The risk constraints are formulated as CVaR constraints, Rockafellar and Uryasev, 2002. The importance of CVaR, among others, is that it is convex when shortfall is convex and linear when shortfall is linear and, hence, the model is easily solvable by linear programming, meaning thereby that very large scenario trees can be handled; one can minimize a weighted sum of CVaR constraints, which is also useful in getting initial feasible solutions when there are several constraints with regard to downside risks; it measures the expectation on the tail and so extremes are handled well; multiple CVaR can be placed on the same distribution and thereby effectively shaping it; many CVaR constraints can be handled by the solvers because usually only a few constraints are active at the optimum (this is why we can handle 21 risk constraints simultaneously); and CVaR is a coherent risk measure. This last technical term means that the measure is sub-additive and is, therefore, useful when aggregating risks measured by outsourcing asset management to different institutions and when risk budgeting, which is usually the case for central banks. VaR and semi-variance are not sub-additive, for instance.

7. Analysis and Insight

Given the model and the tree, we can get an optimal solution. In the case of using several risk constraints this can be done by successively incorporating the risk constraints. An initial model with

21 risk constraints can easily be infeasible and there are relatively simple heuristics for getting an initial feasible solution and successively tightening it by applying constraints.

The objective function we use is to optimize the end-of-period level of reserves in the composite currency that is indicated in Table 3. It should be pointed out that with so many risk constraints the actual feasible region left over for optimizing over naturally becomes relatively small. Much of the solution process is dependent on the tightening of the risk constraints but the optimal value function has important other uses in measuring the marginal value of assets, state prices, and implied prices of risk constraints.

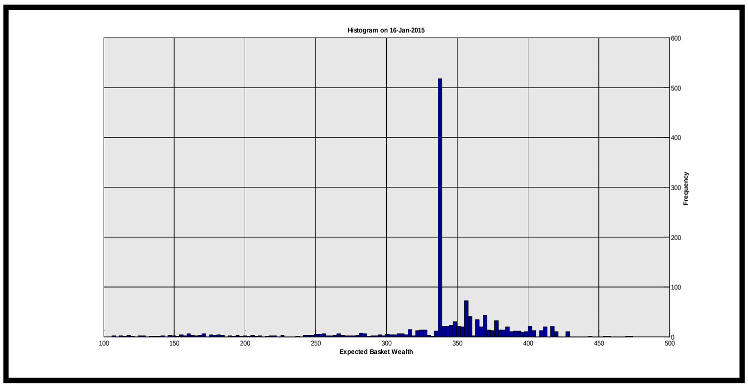
We include a number of tables with considerable information. We think it is useful to do so in order to give an indication of the magnitudes involved and what can be obtained using this framework.

Table 3: Solution Summary

Item	16-Jan 2009	16-Jan 2010	16-Jan 2012	16-Jan 2015
Total Expected Value in US$ Billions	242.63	247.36	284.92	361.08
Total Expected Value in Composite Currency Billions	237.92	242.72	282.00	360.15
Total Expected Value in Euros Billions	185.73	193.49	233.86	208.48
Expected Annualized Return in US$ % over period		1.95%	7.40%	8.43%
STD of Return in US$ % over period		3.41%	12.30%	13.91%
Probability. Return > 2*Expected Return		0.18%	11.77%	5.15%
Probability Return < .5*Expected Return		21.22%	19.72%	11.23%
Probability Return < .1*Expected Return		21.22%	0.54%	2.04%
5% CVaR Value in composite currency		210.72	252.97	330.58
95% CVaR value in composite currency		255.41	323.06	436.04

The Table 3 gives the values of the expected reserves in US dollar, Euro, and the composite currency along with statistics on the distributions. The expected returns increase at the same time the volatility increases. However, as we examine the CVaR values in the composite currency, we see that the distribution is asymmetric more and more on the upside as time goes by. This is seen in the histogram for the distribution of reserves in 2015 in the composite currency in Figure 1.

Figure 1: Distribution of reserves in the composite currency in 2015



We notice the sharp cutoff in the distribution on the downside, which is the result of risk constraints that only penalize the downside. Similar asymmetric distributions hold for the years 2010 and 2012 but less dramatically. These asymmetric distributions also hold for the measurement of the reserves in US dollar, Euro, and other currencies.

In the following Table 4 we provide the expected allocation of assets by currency. The column titled "Composite Currency" gives the relative percentages of each currency in the composite and is based on imports and external debt. In this model, the composition is fixed over time, but in general it can just as well vary over time and with respect to level of other factors.

The expected share of US dollar in the composition is high in part because we assume that the deposits in the BIS and with other central banks are in US dollar. If we had the information on the composition of these deposits the overall percentages will change as will the likely percentage composition of the Euro. Part of the reason for the declining percentage of the Euro is because over the longer term the US dollar is assumed to appreciate with respect to the Euro in the estimation. We can separate out the returns due to currency appreciation versus asset appreciation but do not do so here given the limitation of the size of this paper. We limit the expected share of each of the following three currencies viz. Australian dollar, Pound sterling, and Japanese yen to 15% of the portfolio. The share of Pound sterling, whose was low in 2009, will reach that limit in 2010 and 2012 because the estimation of its exchange rate gives an appreciation due to mean reversion. The model suggests that Pound sterling be bought in 2010. The model was calibrated in January of 2009 and no adjustments were made to the model between then and the publication of this paper.

Table 4: Composite Currency, Initial Portfolio, and Expected Percent Currency Composition (%)

Currency	Composite Currency	Initial Portfolio	16-Jan 2009	16-Jan 2010	16-Jan 2012	16-Jan 2015
Australian dollar	5	2.89	4.15	12.96	13.71	6.57
Euro	20	11.56	1.81	7.55	5.42	9.42
Pound sterling	5	2.89	3.28	15.00	15.00	12.37
US dollar	65	79.77	88.55	63.04	65.55	60.77
Japanese yen	5	2.89	2.06	1.46	0.32	10.88

The column title "Initial Portfolio" gives the currency composition before selling and purchasing other assets. Following those transactions, which were limited to 25% of the portfolio, we have the currency composition indicated in the column of 2009. The main transactions of 2009 were to decrease the Euro composition and to increase the share of US dollar.

The Table 5 gives the detailed transactions for all the asset classes. The main position in 2009 was to move some funds out of BIS and central bank's deposits, Euro, and some yen and move into US corporate bonds, short-term US Treasuries, and yen cash. The column "Average RoR" gives the expected average rate of return over the six year period for each asset class. The "Average downside ratio" gives an equivalent Sharpe Ratio for asymmetric returns and is discussed in Ziemba and Ziemba, 2007. These values tend to be small because we used an expanded volatility measure to generate the trees and to capture the possibilities of more extreme events. The "Marginal Return" column, when negative, gives the expected composite currency loss in the event of selling one unit of the asset class as measured in the currency of origin. When positive, it gives the expected composite currency gain when an additional unit of the asset class is purchased. When the asset units are rescaled to the composite currency, it gives a method of ranking the relative asset classes; that is, which ones are more important to focus on in improving long-term returns.

In the following Table 6, we analyze information on the risk constraints. There are seven risk constraints for each rebalancing period giving a total of 21 risk constraints all cast as CVaR constraints.

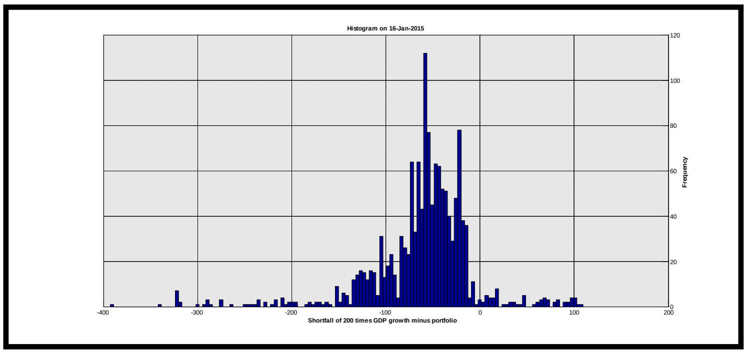
Table 5: Initial Portfolio Rollover analysis (Denominated in currency of origin: Billions)

Rollover actual 25% Rollover limit 25%

Instrument	Sell	Buy	Hold	Percent on Jan 16 2009	Average RoR	Average downside ratio	Marginal return Jan 16 2009
Australian cash total return index		4.37	14.37	4.16%	5.61%	1.62	-0.007
BIS and central banks deposits in US$	22.21		80.34	33.11%	4.89%	0.21	0.01
US$ cash	9.13				1.54%	0.25	0.01
Lehman Global: US Treasury 1-3 Yr		12.85	67.63	27.87%	3.95%	0.25	-0.01
Lehman: US AGG A+ 5+ Yr		0.42	18.69	7.70%	6.01%	0.20	-0.01
US corporate bonds		39.40	48.53	20.00%	8.17%	0.34	-0.01
Euro cash	2.15				1.98%	0.49	0.013
Lehman: Euro AGG. 1-3 Year (E)	12.90				-0.99	-0.03	0.013
Lehman: Euro AGG. 10+ Year (E)	0.93		3.37	1.82%	2.51%	0.01	0.013
Euro corporate bonds					5.72%	0.28	0.013
UK cash		0.64	1.12	0.68%	3.63%	1.17	-0.014
Lehman Global: UK 1-3 Year			2.89	1.74%	4.51%	0.29	-0.013
Lehman Global: UK Long			0.96	0.60%	4.46%	0.06	-0.011
UK corporate bonds			0.48	0.29%	6.49%	0.23	-0.005
Yen cash		330.29	392.85	1.82%	0.02%	0.0	-0.0001
Lehman Global: Japan 1-3 Year	375.35				0.53%	0.002	0.0001
Lehman Global: Japan Long	125.12				2.54%	0.026	0.0001
Yen corporate bonds	9.82		52.74	0.24%	1.38%	0.05	0.0001

We begin by explaining the risk constraint (1) that constrains the risk that the shortfall in reserves below a value of US$ 200 billion times the rate of growth of GDP is small. This smallness is measured as a CVaR constraint that indicates that the expected shortfall in reserves should not be below 5% of the desired growth in reserves with a confidence level of 98%. In Table 6 we measure the probability of any shortfall and that is always below 1%. We can pick different confidence levels and different shortfall percentages for different periods of time but do not do so here for ease of exposition. The CVaR constraint value is the expected value of the growth in the desired lower level that is the constraint level. The last item is the annualized marginal return in the expected value of the portfolio with a 1% change in the confidence level of the constraint. It gives an indication of how much the expected value of the portfolio as measured in the composite currency will change when the confidence level is adjusted by 1%. When the value is zero it means the risk constraint is not binding.

Figure 2: Histogram of shortfall in reserves to growth level for 2015



The histogram of the shortfall is given in the following Figure 2. Note the concentration below zero, i.e. no shortfall. And the amount of shortfall above zero is very small; as we saw in the table, less than 1%.

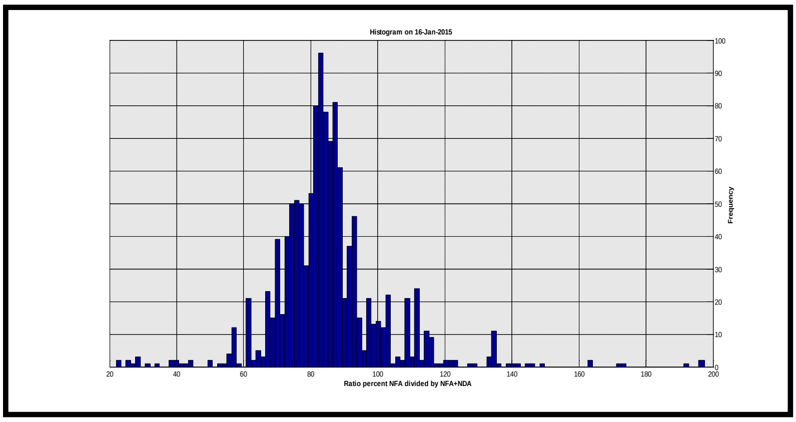
Several additional measures for these constraints can be discussed here but that is beyond the scope of this paper. An important one is the measure of state prices generated by the risk constraints on each scenario. These can be used to measure the relative importance of scenarios and to identify those scenarios that impact other risk constraints and how the asset variables impact through the risk constraints.

Table 6: Risk Constraints

Constraint Shortfall Item	16-Jan-2010	16-Jan-2012	16-Jan-2015
(1) US$ Billion 200 times GDP growth: shortfall expectation is 5% below expected with a 98% confidence level			
Expected Value of growth	208.15	238.72	298.74
Probability of a shortfall	0.35%	0.35%	0.43%
CVaR constraint value	197.74	226.31	283.81
Annualized marginal return of 1% change in confidence	0.0%	0.0%	0.0%
(2) Ratio of NFA/(NFA+NDA): shortfall expectation is 10% below expected with a 95% confidence for a 50% ratio			
Expected value of NFA in US$ billion	247.37	284.92	361.08
Probability of a shortfall	0.81%	0.73%	0.26%
CVaR constraint value	222.63	256.43	324.97
Annualized marginal return of 1% change in confidence	0.027%	0.000007%	0.002%
(3) Liquidity at risk- Liquid assets less requirements: shortfall expectation is 5% below at a 99% confidence			
Expected liquid assets in US$ billion	137.28	207.47	317.40
Probability of a shortfall	0.65%	5.01%	0.45%
CVaR constraint value	130.42	197.09	301.55
Annualized marginal return of 1% change in confidence	0.0%	0.004%	0.0%
(4) Stability of reserves in terms of US$ returns: shortfall expectation is 20% below expected at 99% confidence			
Expected annualized return in US$	1.95%	7.40%	8.43%
Probability of a shortfall	21.22%	0.53%	1.83%
CVaR constraint value	1.56%	5.96%	6.85%
Annualized marginal return of 1% change in confidence	0.0%	0.006%	0.008%
(5) Market limit loss in composite currency: shortfall expectation is 20% below expected with a 90% confidence			
Expected value of composite currency in billions	242.72	282.00	360.15
Probability of a shortfall	59.71%	61.22%	60.70%
CVaR constraint value	218.45	253.80	324.14
Annualized marginal return of 1% change in confidence	0.0%	0.001%	0.502%
(6) Foreign currency in excess of amortized external debt: shortfall is 10% below expected with a 95% confidence			
Expected value of assets in US$ billion	247.37	284.92	361.08
Probability of a shortfall	0.81%	0.73%	0.26%
CVaR constraint value	222.63	256.43	324.97
Annualized marginal return of 1% change in confidence	0.027%	0.000007%	0.002%
(7) Ratio of short-term-debt to reserves: shortfall is 15% below expected with a 92% confidence for a ratio of 40%			
Expected value of reserves in US$ billion	247.37	284.92	361.08
Probability of a shortfall	40.81%	24.78%	15.78%
CVaR constraint value	210.26	242.18	306.92
Annualized marginal return of 1% change in confidence	0.054%	0.0%	0.004%

The risk constraint (2) limits the size of the ratio NFA/(NFA + NDA). The ratio limit is 50%. This limit reduces to NFA $\leq$ NDA. The probability of any shortfall in this ratio is very small (less than 1%) but the constraint is binding in all periods. The confidence level on this measure is 95%, however, a change in that confidence level in 2010 of 1% will amount, at the margin, to over US$ 6.6 billion. So it is very significant.

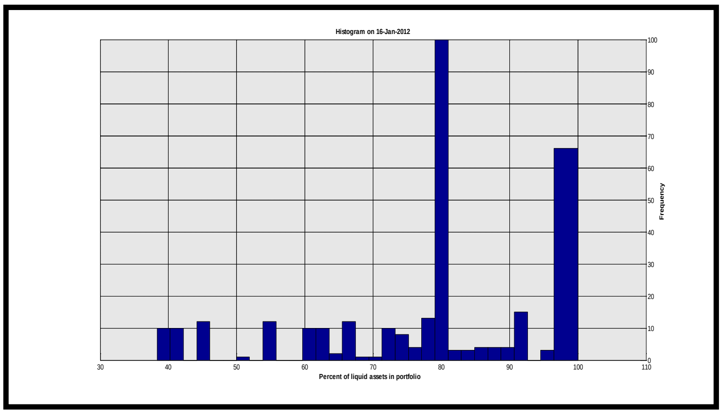
Figure 3: Histogram of shortfall in ratio of NFA/(NFA+NDA) for 2015



Again we can see the sharp cutoff in the histogram below 50%. There are a few outliers below and above that we choose not to illustrate so as to not distort the diagram. But the probability of a shortfall is very low.

The next constraint (3) is the 'liquidity at risk'. The liquidity requirements can be estimated in several ways, including using contingent claims, 4. Again the probability of a shortfall is small and only the risk constraint in 2012 is binding. But the requirements are very tightly imposed at at 99% confidence level and a 5% shortfall. The liquid assets include cash and short term bonds in all the currencies along with the BIS and central banks deposits so this value is fairly high.

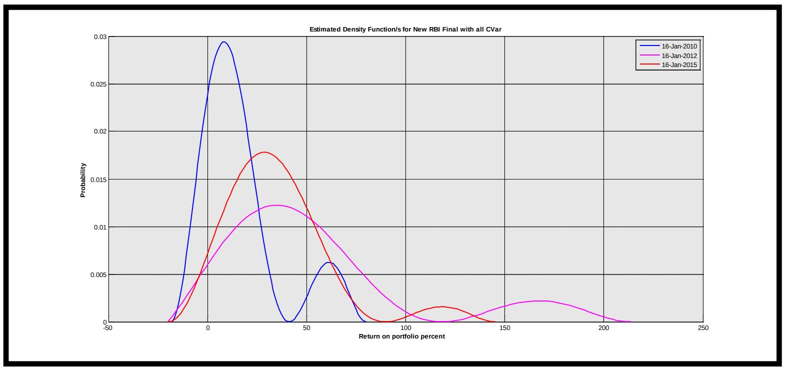
Figure 4: Histogram percent of liquid asset in portfolio for 2012



However, even when we measure the percentage of liquid assets in the portfolio as seen in Figure 4, we see that there is a considerable variation in the levels. This is of course because of the varying level of liquid asset requirements in some scenarios when the level of the assets is high. The detailed analysis is beyond the scope of this paper.

The next constraint (4) constrains the risk of a percentage loss in the US$ value of the portfolio. The probability of a shortfall is high in 2010 at 21.22% but the CVaR value at 1.56% is not binding. If we graph the densities over time as in Figure 5, we see how the upper tail moves to the right over time but the lower part is cut off at around a loss of -20%.

Figure 5: densities of portfolio returns over all periods



But as we move out in time the portfolio returns improve with the process estimation expectation reverting back to a more normal situation.

The next risk constraint (5) is the market limit loss in the composite currency. This constraint is only binding in 2012 and 2015 but the value of the annualized marginal return shows that this constraint has the greatest impact of any of the risk constraints considered here (a value of 0.502%). This is very significant but in order to maintain good risk control when the volatility is higher, we need very tight constraints on the market loss in the future especially as to that one that is driven by the composite currency, which is the driving force in the objective. The Figure 6 shows the sharp cutoff in the shortfall at around 20%, which is the constraint value. But at a 90% confidence level you have several outliers. We should mention that it was difficult to increase the confidence level of this constraint without giving up some other risk constraint as we do experience some extremes in the shortfall when measured in the composite currency.

Figure 6: Market limit loss in composite currency in 2015

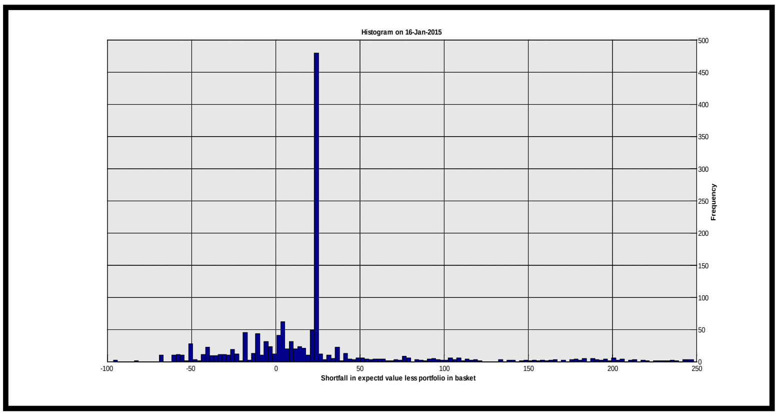
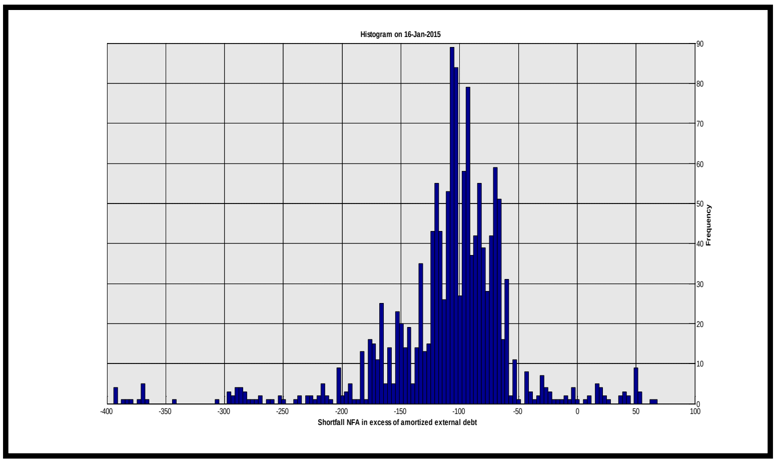


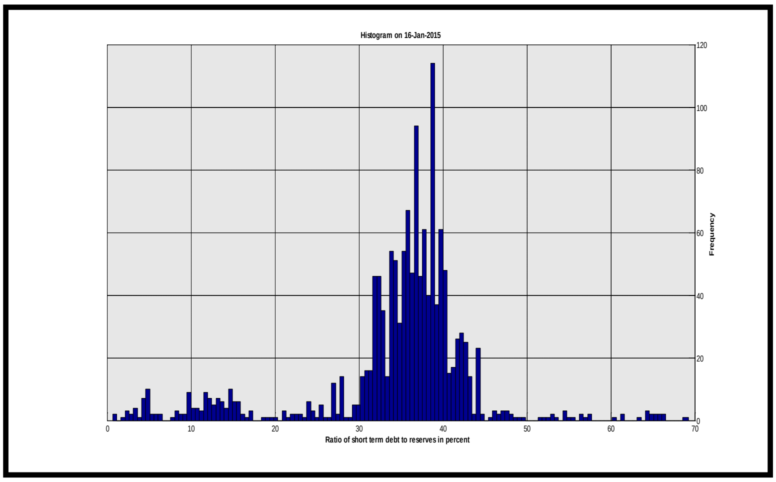
Figure 7: Foreign currency in excess of amortized external debt in 2015



The constraint (6) measures the foreign currency in excess of amortized external debt. The shortfall is limited to 10% below the expected with a 95% confidence level. Again the Figure 7 shows the sharp cutoff in the shortfall below zero. The risk constraint is binding in every period and the histograms look similar to this one.

The last risk constraint (7) is for the ratio of short-term-debt to reserves with the shortfall at 15% with a confidence level of 92% for a ratio of 40%.

Figure 8: Ratio of short-term-debt to reserves in 2015



The histogram in Figure 8 is for 2015 but it looks the same in most every period even though the risk constraint is only binding in 2010 and 2015. You can see the sharp cutoff at around 40% and then several outliers. But in most case the probability of any shortfall is pretty substantial.

In the following Table 7 we tabulate the reserves as a percent of GDP. This has not been entered into the model as a risk constraint but we include it here so as to indicate the results mentioned in Wyplosz, 2007, where he discusses the use of the optimal reserves as a percentage of GDP as an indicator of self-insurance. We already have self-insurance indicators in the form of the lower limit on the rise in reserves value with respect to the growth rate of GDP and on the limits with respect to short-term-debt and on amortized external debt. As can be seen in the table, the ratio is very stable with respect to a ratio around 22%, which is consistent with measures in Asia around 2002.

Table 7: Reserves as a % of GDP

	16-1-09	16-1-10	16-1-12	16-1-15
5% VaR		21.59	20.18	20.02
Mean		22.34	22.47	22.70
95% VaR		22.68	27.24	26.64

One last issue we want to discuss here and that is the opportunity cost of each currency in the portfolio and that information is given in Table 8. The opportunity cost of the US dollar is close to the average return on the portfolio as a whole. The Pound sterling marginal return is high because, in part, of the mean reversion of the exchange rate over time.

Table 8: Opportunity costs of currencies

Currency	Marginal return in composite currency
Australian dollar	3.03%
Euro	14.22%
Pound Sterling	16.40%
US dollar	9.31%
Japanese Yen	-48.51%

The Euro return is also high but not as much Euro appears in the solution because of the risk constraints that tend to be biased toward the US dollar. Further analysis of this table is given in Bhattacharya et al, 2010. Several other aspects of the analysis of the solution and a further discussion of the kinds of analysis that can be done are discussed in the same paper.

8. Integration into the RBI Risk Culture

This framework can be seamlessly integrated into the existing risk culture because of the ease with which multiple objectives are handled within it. Several factors make this integration more natural and they include: (i) Models that are tailored to institutional objectives and constraints; (ii) Combining multi-factor risks; (iii) Opening up the possible menu of risks that can simultaneously be incorporated; (iv) Providing multi-factor stress tests with bundles of different macro-economic and micro-financial environments lumped together; (v) Integrating expert opinion either in the form of scenarios, kinds of events, or statistical attributes of economic or financial processes such as the direction of volatility of an exchange rate, crisis indicators, etc.; (vi) Incorporating herding, correlation convergence, or concentration risks; (vii) Incorporating theories or implied pricing information; and (viii) Easily fostering internal debates on risks and methods of resolution.

Some of the direct benefits in analysis for the RBI include undertaking multi-factor stress tests; Having a very flexible and easily updated structure promoting *ad hoc* analysis; Using at the instrument, portfolio, aggregated institutional, or sovereign level; Using very open, flexible, understandable, and easy to modify models; Analyzing sudden stops in currency flows; Incorporating extreme events; and Satisfying the principles for sound stress-testing practices.

India is outsourcing some reserves to external managers but could pursue it on a larger scale. John Nugee, head of the official institutions group at State Street Global Advisors, emphasizes that central banks with very large reserves need to approach the outsourcing issue with a view towards building solid long-term relationships, as the traditional reserves management is not scalable. The transitional model has been to augment the knowledge of external managers, who can provide their views on markets, skills transfer, and staff training. The key decisions facing the central bank are asset allocation, liquidity, currency composition, risk/return, objectives for each asset class, active versus passive strategy, the cost of management/number of managers, the ability of the manager to handle large assets, and the expertise of the manager to exploit market inefficiencies. This framework can play a significant role in assisting that allocation process.

9. Summary

We have demonstrated a framework that can satisfy many critical requirements of the Reserve Bank of India and incorporate multiple risk factors that can be used to stabilize the strategic asset allocation of their reserves. In particular, we have incorporated several factors that can be used to provide self insurance in the event of financial and economic shocks. The framework is open, flexible, tailored, and comprehensive yet provides a natural framework that should be able to be understood, analyzed, and implemented by any central bank.

Annex: Model Equations

The object functions we used here is the maximization of the end-of-period level of expected reserves in the basket currency.

$$(2) \qquad \text{MAX} \quad \sum_{t \in T} \delta^t W^t \qquad \delta^T = 1, \quad \delta^t = 0 \forall t < T$$

The cash flow equations immediately provide the balance sheets at any time *t* and event *e* or the expected balance sheet at time *t*.

Cash Flows: By currency, time, and event

$$\underbrace{\sum_{i \in I} A_{i,c}^{t,e,t,e}}_{\text{[new assets]}} + \underbrace{\sum_{i \in I} tca_i A_{i,c}^{t,e,t,e}}_{\text{[transaction costs on new assets]}} + \underbrace{V_{c,d}^{t,e}}_{\text{[transfer of currency } c \text{ to } d]} + \underbrace{CASH_c^{t,e}}_{\text{[cash in currency } c]}$$

$$= \underbrace{\sum_{i \in I} ina_{i,c} \langle \text{if } t{=}0 \rangle}_{\text{[initial portfolio]}} + \underbrace{\sum_{d \in C, d \neq C} \frac{tc_c \gamma_c^{t,e} V_{d,c}^{t,e}}{\gamma_d^{t,e}}}_{\text{[transfers of currency } d \text{ to } c]}$$

$$(3) \qquad + \underbrace{\sum_{i \in I} \sum_{\substack{(\tau,\varepsilon) \in AT^{t,e} \\ \tau \geq t - mat_i}} \sum_{(t-1,f) \in AT^{t-1,f}} (1 + tcs_i) \eta_{i,c}^{\tau,t,e} \left(A_{i,c}^{\tau,\varepsilon,t-1,f} - A_{i,c}^{\tau,\varepsilon,t,e} \right)}_{\text{[asset sales]}}$$

$$+ \underbrace{\sum_{(t-1,\varepsilon) \in AT^{t,e}} (1 + rc) CASH_c^{t-1,\varepsilon}}_{\text{[return on cash]}}$$

$$\forall\ c \in C,\ t \in T, e \in X^t$$

The next equation ensures that the sales of assets are properly defined in the model, i.e. to be monotonic.

Monotone Asset Sales

(4) $$\sum_{(t-1,f)\in AT^{t,e}} \left(A_{i,c}^{\tau,\varepsilon,t-1,f} - A_{i,c}^{\tau,\varepsilon,t,e} \right) \quad \geq \quad 0$$

[monotonicity constraint so that sales are reflected in changes in holdings]

$$\forall \; i,c,t,e,\tau,\varepsilon \;\ni\; (\tau,\varepsilon)\in AT^{t,e} \text{ and } (t-\tau) < mat_i$$

Total Wealth: In numeraire currency.

(5) $$TW^{t,e} = \sum_{i\in I,c\in C} \frac{A_{i,c}^{t,e,t,e}}{\gamma_c^{t,e}} + \sum_{i\in I,c\in C} \sum_{(\tau,\varepsilon)\in AT^{t,e}} \frac{\eta_{i,c}^{\tau,t,e} A_{i,c}^{\tau,\varepsilon,t,e}}{\gamma_c^{t,e}}$$

[total assets] [new assets] [assets marked to market]

$$+ \sum_{c\in C} \frac{CASH_c^{t,e}}{\gamma_c^{t,e}}$$

[cash] $\forall \; t,e \;\ni\; e\in X^t$

The following defines the total expected value of the portfolio as measured in the numeraire.

Total Expected Wealth Definition

(6) $$W^t = \sum_{e\in X^t} \pi^{t,e} TW^{t,e} \qquad \forall \; t\in T$$

Asset Sales: By individual asset in time *t* and event *e*.

(7) $$SL_{i,c}^{t,e} = \sum_{\substack{(\tau,\varepsilon)\in AT^{t,e} \\ \tau\geq t-mat_i}} \; \sum_{(t-1,f)\in AT^{t-1,f}} \eta_{i,c}^{\tau,t,e} \left(A_{i,c}^{\tau,\varepsilon,t-1,f} - A_{i,c}^{\tau,\varepsilon,t,e} \right)$$

Maximum Portfolio Rollover:

(8) $$\sum_{(i,c)} \frac{SL_{i,c}^{t,e}}{\gamma_c^{t,e}} \leq percentrollover^t TW^{t,e}$$

Market Activity Limits:

(9) $$A_{i,c}^{t,e,t,e} + \sum_{(\tau,\varepsilon)\in AT^{t,e}} \eta_{i,c}^{\tau,t,e} A_{i,c}^{\tau,\varepsilon,t,e} \leq upperlimit_{i,c} TW^{t,e}$$

[new assets] [assets marked to market]

CVaR Constraints can be defined by time period, activity, confidence level, and level value and multiple constraints can be included.

(10)
$$Z^e \geq \Lambda\left(X^e\right) - ALPHA, \quad Z^e \geq 0, \quad \forall e \in X^t, \text{ and for some } t$$
and
$$\sum_{e \in X^t} \pi^{t,e} Z^e \leq \rho\left(clevel_\rho - ALPHA\right)$$

Where Λ is a convex loss function, Z^e and $ALPHA$ are variables, $1-\rho$ is the confidence level, and $clevel_\rho$ is the value based upon that confidence level.

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[5] In RBI, 2009, Dr. D. Subbarao, Governor, Reserve Bank of India in the Annual Policy Statement[5] 2009-10.

[6] Mohan, Rakesh, 2009.

[7] See Zenios and Ziemba, 2007 and Ziemba and Ziemba, 2007.

[8] A binary tree of these 27 factors over three periods would be so difficult to solve that if we had a super computer to evaluate one million scenarios per second the entire tree could not be evaluated over the lifetime of our existing universe. Let alone it would not capture extreme events.

[9] Given the space limitations, we do not discuss the details of the data estimation here. That can be found in Bhattacharya et al, 2010.