 

Busting Bubbles

Applications of a New Rational Expectations Bubble Model

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Based on a paper coauthored with Didier Sornette: ETH Zürich

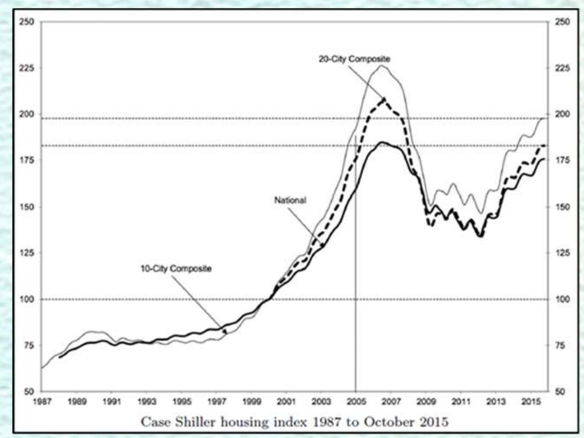
The Motivation

Black Swan Essence (Nassim Taleb)

1. Surprise event - rare
2. Major impact
3. Retrospective analysis

No predictability

Probabilistic predictability



Dragon King Essence (Sornette)

1. Unsustainable feedback
2. Correction
3. Probabilistic prediction

Most bubbles

The Process – The Idea

1. Prices oscillate around a "normal price"
2. Geometric random walk with jumps
3. Define a model with both elements
4. Estimate jump size and frequency
5. Prices accelerate before a crash
6. Estimate model components
7. Apply to real bubbles and refine

$$p_{t+1} = p_t \exp\left(\bar{a}_t + \sigma\varepsilon_t\right) \quad \text{with} \quad p_0 > 0$$

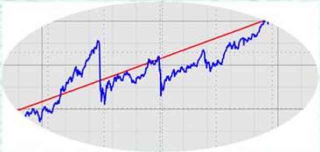
and

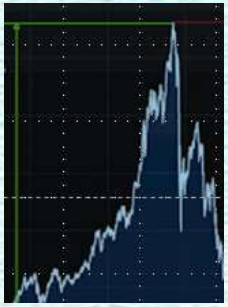
$$\bar{a}_t = \begin{cases} \bar{r}_t & \text{with probability } 1-\rho_t \quad \text{with } 0 \le \rho_t < 1 \\ \kappa_i \ln\left(q_t\right) + r_D & \text{with probability } \rho_t \eta_i \quad i = 1, 2, \ldots, n \\ & \kappa_i \in \Omega \equiv \left\{ \kappa_i \middle| -\infty < \kappa_i < \infty,\ i = 1, 2, \ldots, n \right\} \end{cases}$$

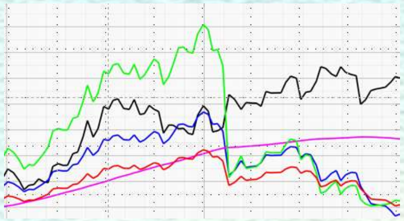
with

$$q_t = \frac{N_t}{p_t} \quad \text{and} \quad \sum_{i=1}^{n} \eta_i = 1 \quad 0 < \eta_i < 1 \text{ and } \bar{K} = \sum_{i=1}^{n} \eta_i \kappa_i$$

$$N_t = p_0 \exp\left(r_N t\right)$$







Keep in mind that it is not a matter of “are we in a bubble”

Rather

<u>What to do when we are in a bubble!</u>

Geometric random walk with jumps

The Model

$$p_{t+1} = p_t \exp\left(\overline{a}_t + \sigma\varepsilon_t\right) \quad \text{with} \quad p_0 > 0$$

and

$$\overline{a}_t = \begin{cases} \overline{r}_t & \text{with probability } 1-\rho_t \quad \text{with} \quad 0 \le \rho_t < 1 \\ \kappa_i \ln\left(q_t\right) + r_D & \text{with probability } \rho_t \eta_i \quad i = 1, 2, ..., n \\ & \kappa_i \in \Omega \equiv \left\{\kappa_i \middle| -\infty < \kappa_i < \infty,\ i = 1, 2, ..., n\right\} \end{cases}$$

with

$$q_t = \frac{N_t}{p_t} \quad \text{and} \quad \sum_{i=1}^{n} \eta_i = 1 \quad 0 < \eta_i < 1 \text{ and } \bar{K} = \sum_{i=1}^{n} \eta_i \kappa_i$$

$$N_t = p_0 \exp\left(r_N t\right)$$

$$E_t\left[\ln\left(\frac{p_{t+1}}{p_t}\right)\right] = \left(1-\bar{\rho}\right)\bar{r}_t + \bar{\rho}\bar{K}\ln\left(\frac{N_t}{p_t}\right) + \bar{\rho} r_D$$

$$= r_D$$ **RE Condition**

$$\bar{r}_t = r_D - \frac{\bar{\rho}\bar{K}\ln\left(q_t\right)}{1-\bar{\rho}}$$

$\bar{\rho}, \bar{\mathrm{K}}$ may be averages or depend on time

Estimating Parameters

We will separate the geometric random walk from jumps and jump probabilities

Estimating Parameters

$$\sigma, \bar{\mathrm{K}}, \bar{\rho}$$

Use Audrino and Hu (2016) to test if r_t is a jump.
Calculate occurrence and size.

ρ_t **Comes later!**

Estimating Parameters

By Optimizing portfolios

$$r_D, r_N, \sigma, \bar{K}, \bar{\rho}$$

Max log of wealth = Kelly to allocate between risky and riskless assets

$$W_{t+1} = \left(\lambda_t \exp\left(\overline{a}_t + \sigma \varepsilon_t \right) + \left(1 - \lambda_t \right) \exp\left(r_f \right) \right) W_t$$

$$\max_{\lambda_t} E_t \left[\ln \left(\frac{W_{t+1}}{W_t} \right) \right] \qquad -1 \leq \lambda_t \leq 2$$

Kelly

Using bubble model – estimating allocation to risky asset

$$\lambda_t^* \approx \frac{r_D - r_f + \dfrac{\sigma^2}{2}}{\sigma^2 + (1-\rho)\left(\bar{r} - r_f\right)^2 + \rho\left(\bar{\mathrm{K}}\ln\left(q_t\right) + r_D - r_f\right)^2}$$

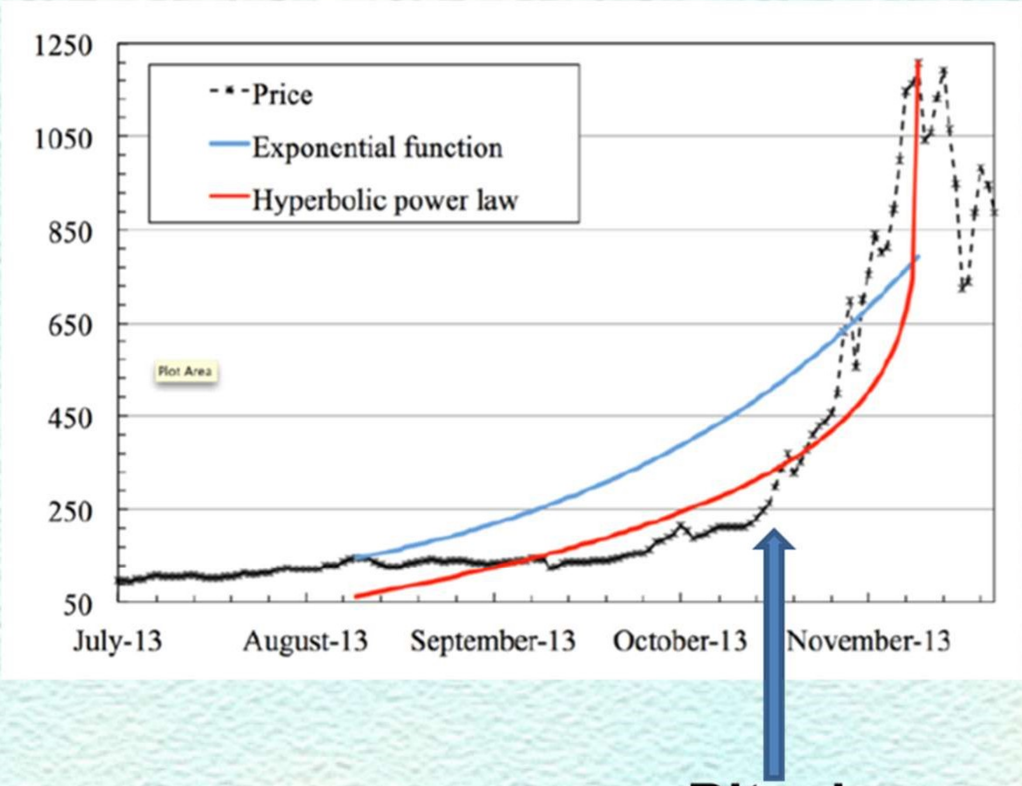
Do prices accelerate before a crash/rally AND probabilities change.

Feedback

Week on the Wild Side – Danger Zone
The Economist Week The Wild Side: November 1997. The cartoon from the Economist. Reproduced by permission of Kevin Kallaugher (Kaltooons).

Acceleration in early Bitcoin crash

Acceleration signatures exist in price data



Bitcoin

Data has a pattern

Example: 1929 Crash

Can we measure acceleration and relate to crash size and probability?
And thusly to risky asset allocation.

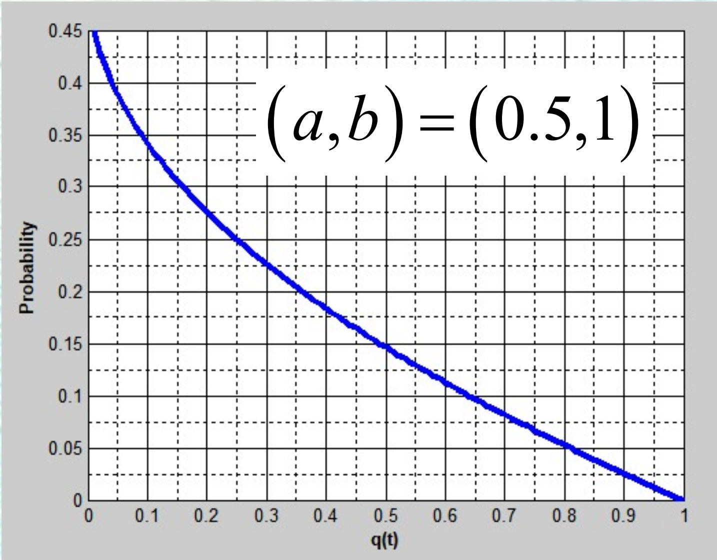
$$\rho(q) \equiv \begin{cases} \dfrac{1-q^a}{1+b} \quad b > -1 & \begin{array}{l} \text{if } q < 1 \Rightarrow a > 0 \\ \text{if } q > 1 \Rightarrow a < 0 \end{array} \end{cases}$$

$$d_t \equiv \ln\left(\frac{p_{t+1}}{p_t}\right) - r_D + \frac{\left(1-q_t^a\right)}{q_t^a + b}\bar{K}\ln\left(q_t\right)$$

Estimate expected excess return

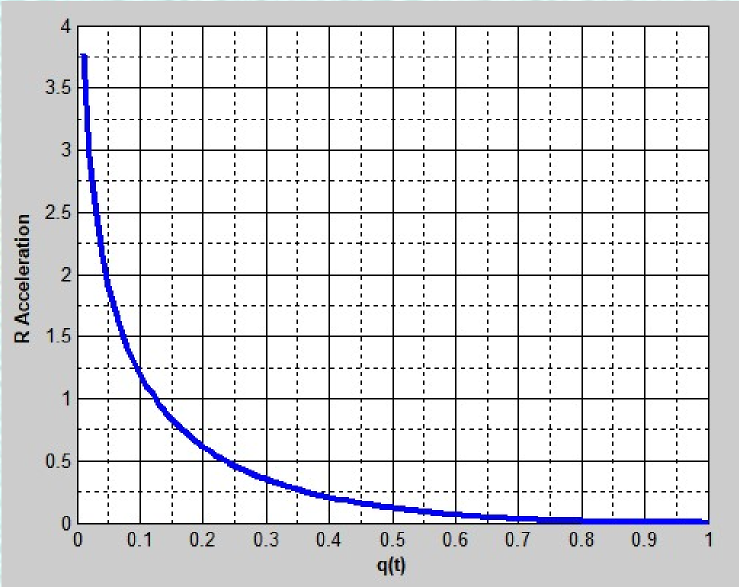
$b < 0$ Super-exponential growth with finite time singularity

$$\rho(q) \equiv \left\{ \frac{1-q^a}{1+b} \right. \quad b > -1 \qquad \begin{array}{l} \text{if } q < 1 \Rightarrow a > 0 \\ \text{if } q > 1 \Rightarrow a < 0 \end{array}$$



$$d_t \equiv \ln\left(\frac{p_{t+1}}{p_t}\right) - r_D + \left[\frac{(1-q_t^a)\ln(q_t)}{q_t^a + b}\right]\bar{K}$$

$$= \ln\left(\frac{p_{t+1}}{p_t}\right) - r_D + R_{a,b}(q_t)\bar{K}$$



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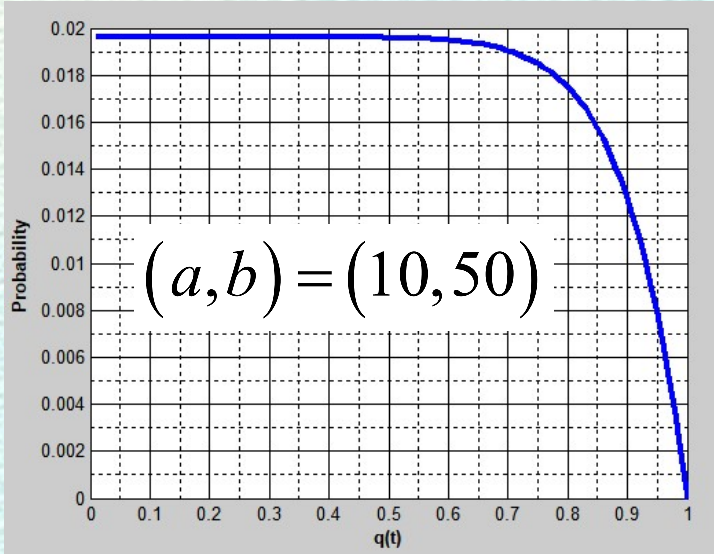

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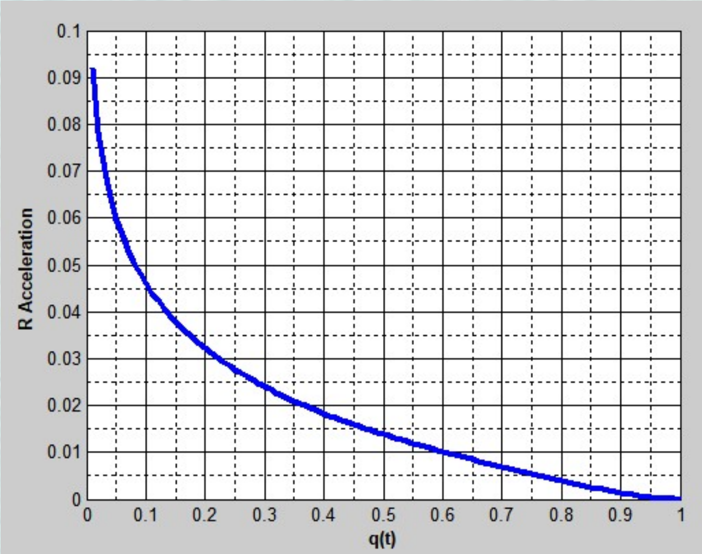

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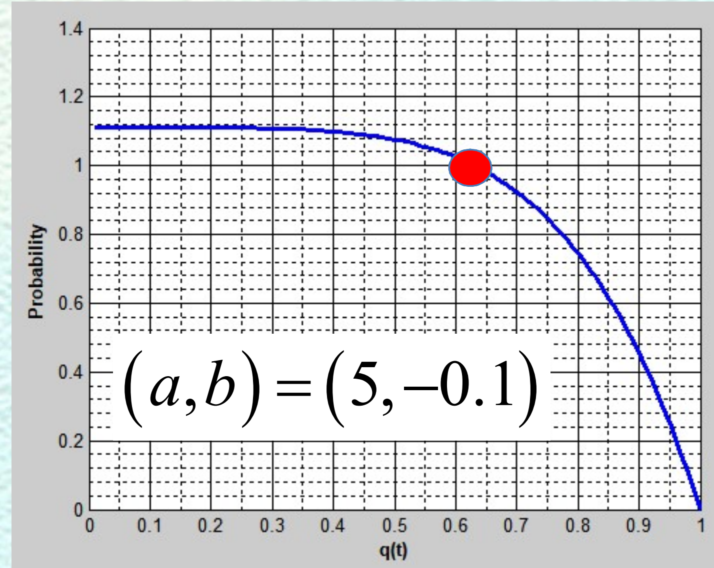

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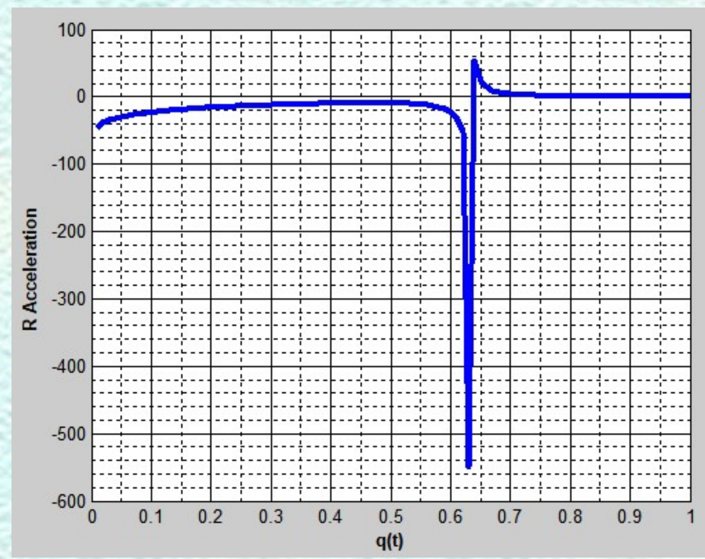
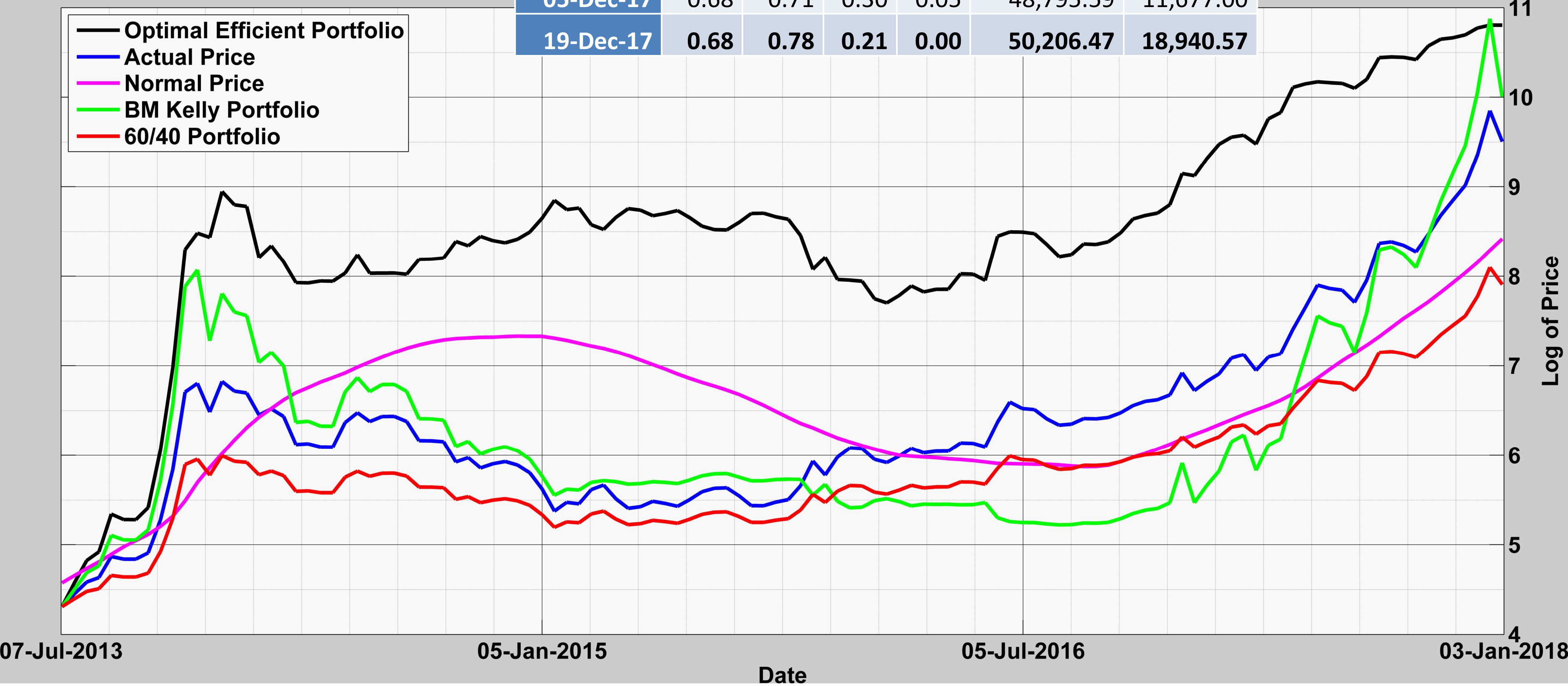


Finite time singularity at $\bar{q}_t = .631$

Bitcoin bubble mitigation Dec 2017

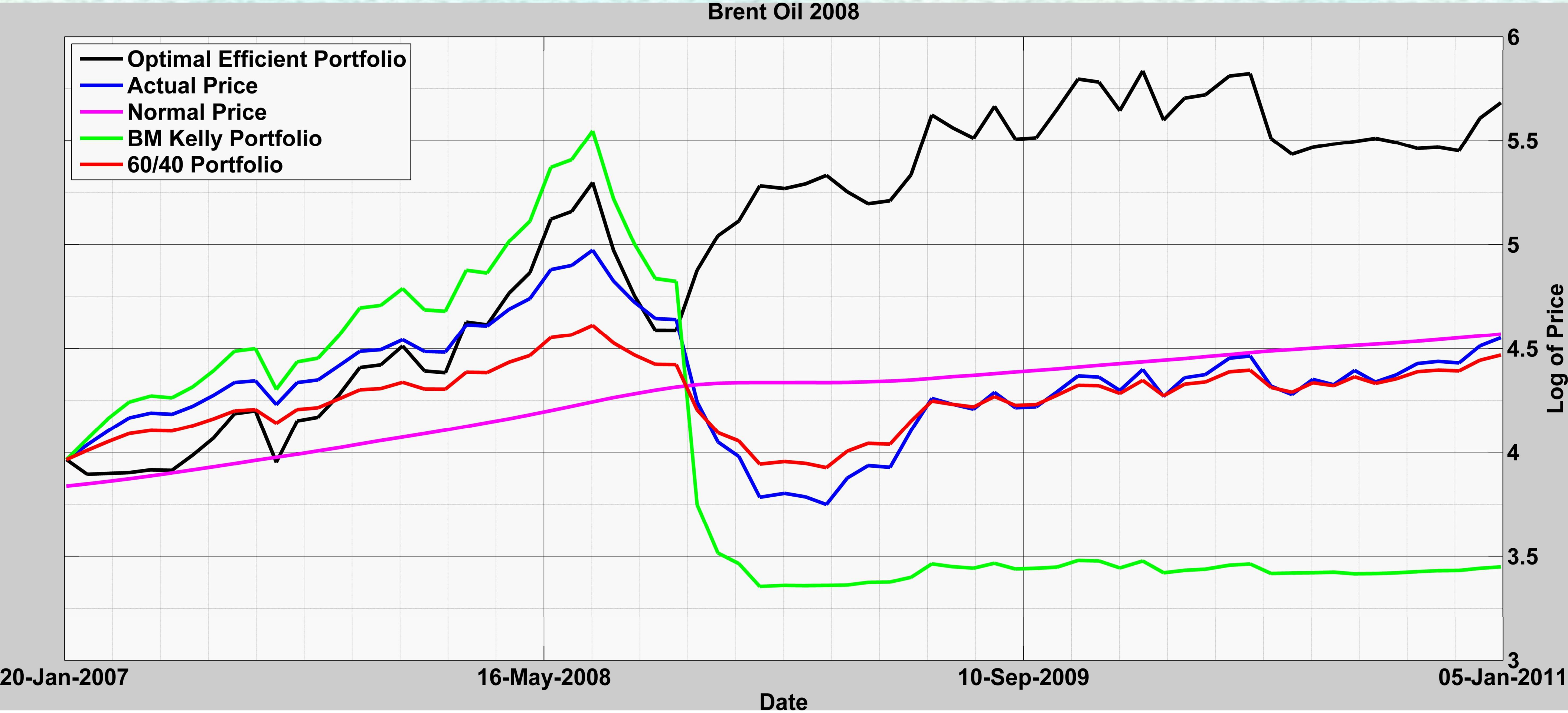
Date	Crash	Prob	qt	Lam	Eff Port	Price
24-Oct-17	1.22	0.73	0.46	0.10	42,950.55	5,523.40
07-Nov-17	1.22	0.64	0.39	0.15	44,239.07	7,130.28
21-Nov-17	1.22	0.61	0.39	0.18	45,159.44	8,095.23
05-Dec-17	0.68	0.71	0.30	0.05	48,795.59	11,677.00
19-Dec-17	**0.68**	**0.78**	**0.21**	**0.00**	**50,206.47**	**18,940.57**

Dec. 19, 2017 predicted to drop to $6,554 with prob = .78.



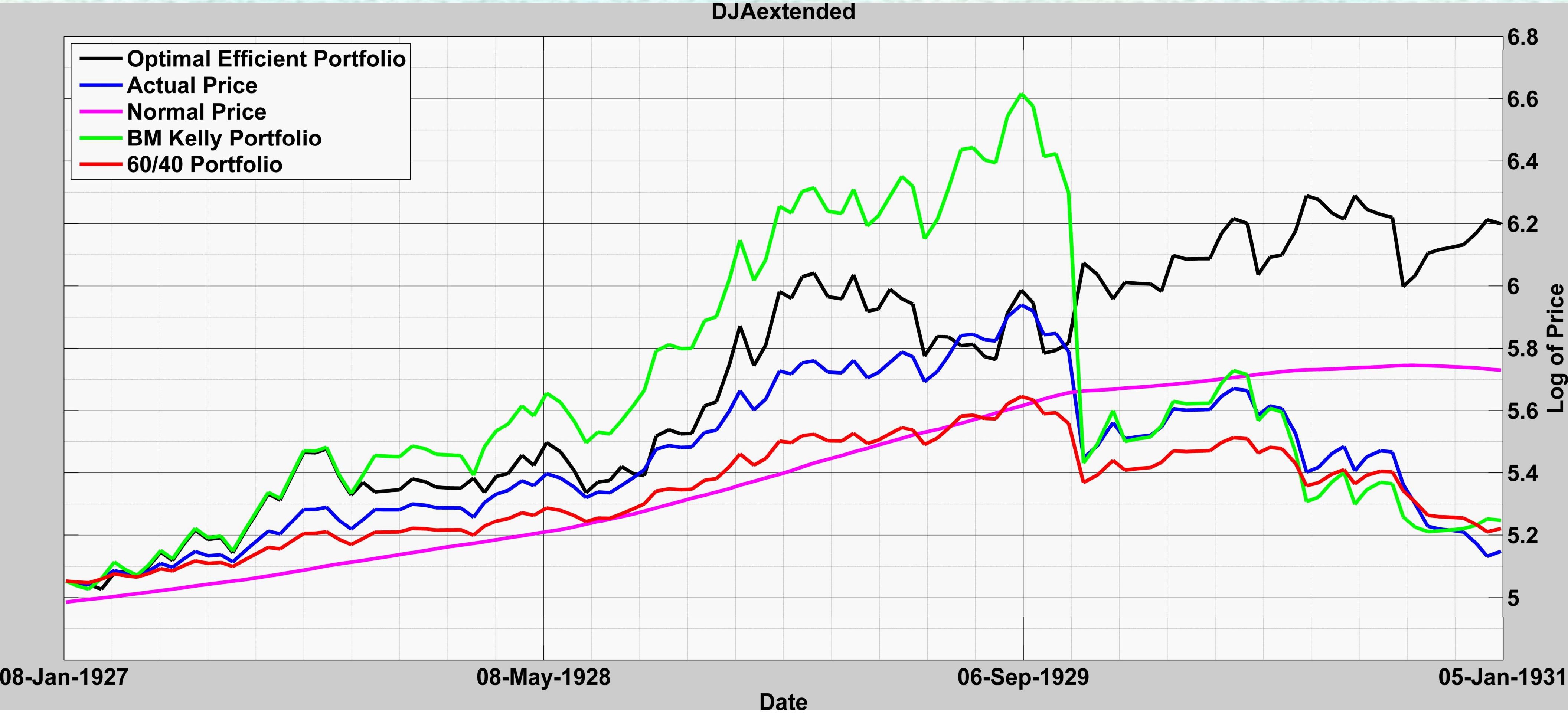
Brent Oil 2008

Fig 2: Brent Oil – 4 Jan 2005 to 3 Jan 2011				
	CAGR	Sharpe	Max Drawdown	Calmar
Actual Price	14.69	0.24	70.57	0.21
Optimal Efficient Portfolio	42.94	0.70	50.85	0.84



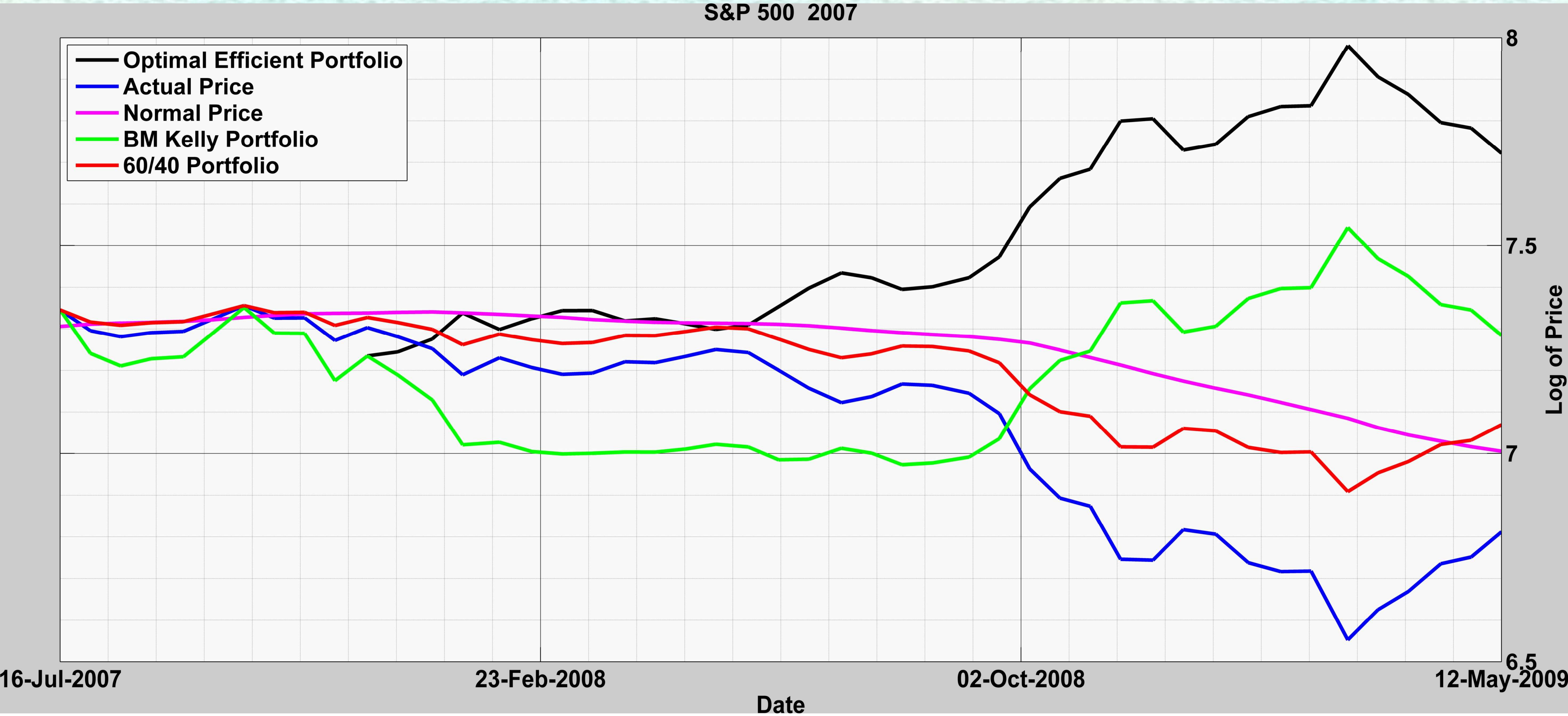
DJ 1929

Fig 3: Dow Jones 1929 – 7 Jan 1927 to 3 Jan 1931				
	CAGR	Sharpe	Max Drawdown	Calmar
Actual Price	2.37	-0.08	55.37	0.04
Optimal Efficient Portfolio	28.65	0.61	25.26	1.13



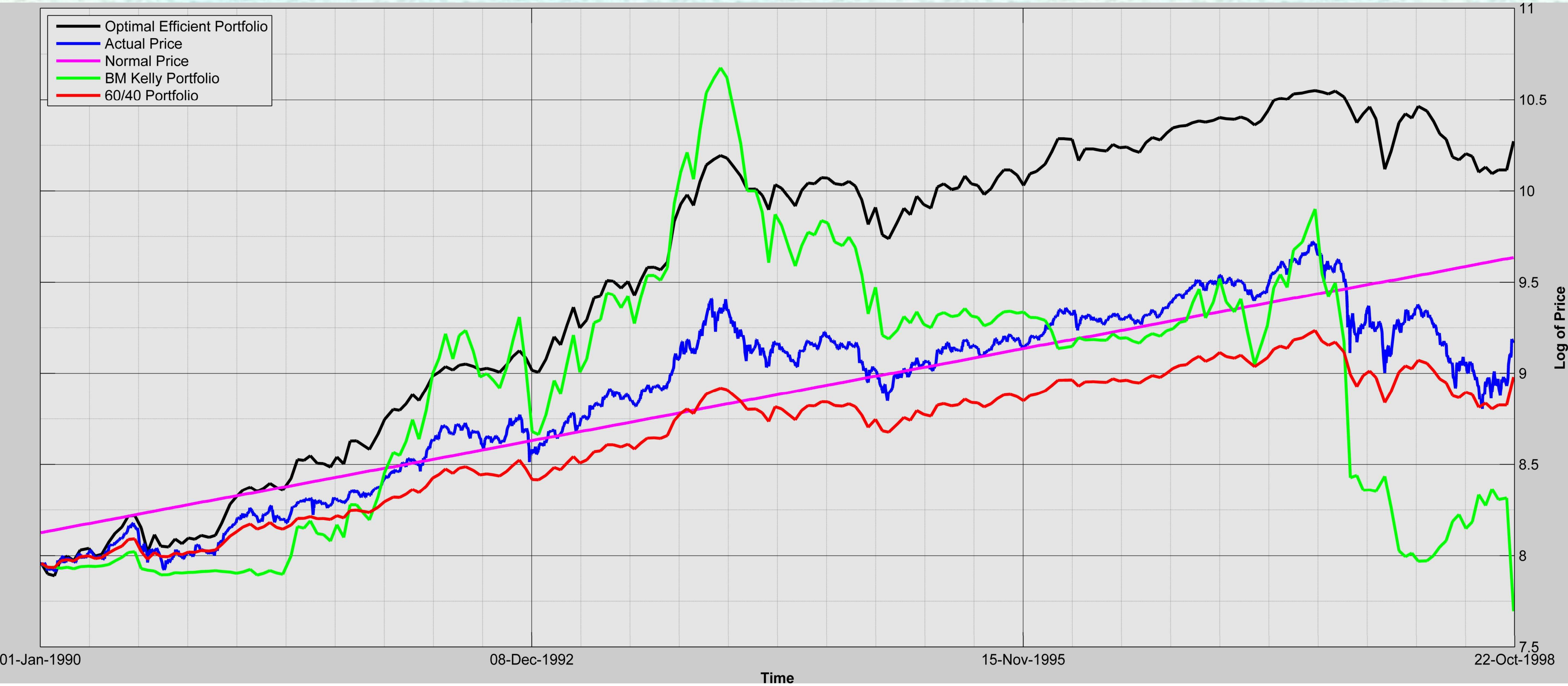
S&P 500 2007

Fig 4: S&P 500 2007 – 2 Jul 2007 to 12 May 2009				
	CAGR	Sharpe	Max Drawdown	Calmar
Actual Price	-28.49	-1.31	55.22	-0.52
Optimal Efficient Portfolio	20.05	0.59	22.83	0.88



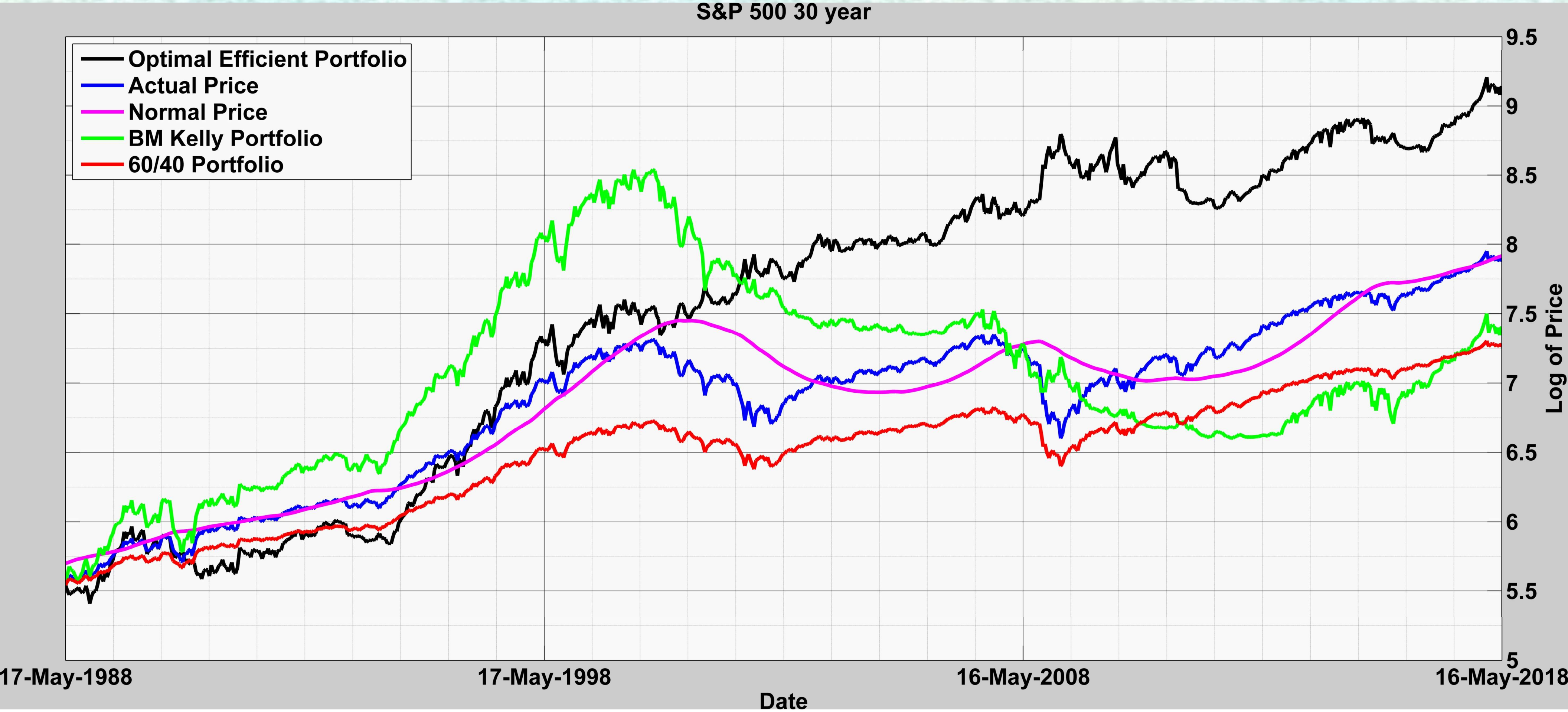
Hong Kong

Fig. 1: Hong Kong – 10 Jan 1996 to 20 Oct 1998				
	CAGR	Sharpe	Max Drawdown	Calmar
Actual Price	-2.37	-0.19	56.40	-0.04
Optimal Efficient Portfolio	35.65	0.70	29.16	1.22



S&P 500 – 30 year

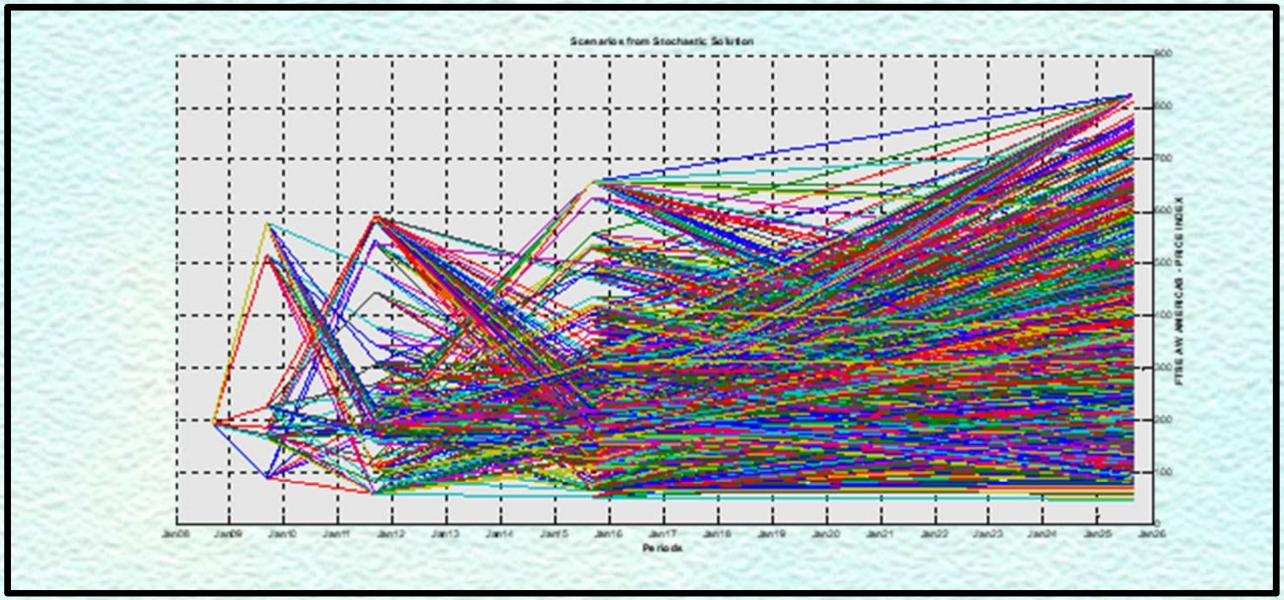
Fig 7: S&P 500 30 years: 17-May-1988 to 16-May-2018.				
	CAGR	Sharpe	Max Drawdown	Calmar
Actual Price	7.89	0.40	52.65	0.15
Optimal Efficient Portfolio	12.00	0.46	41.61	0.29



Integrating into a portfolio …

Enter AugurMax

http://riskontroller.com/augurmax/



That is for another story

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