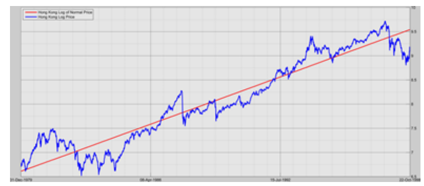
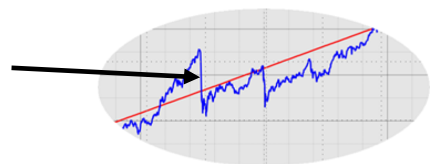
A Short Overview of *ECO*

ECO (Efficient Crashes Optimizer) is our technology applying an innovative asset bubble price model. ECO's theoretical construction is described here.

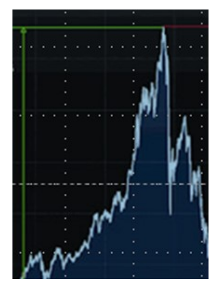
1. **Normal Price**
   The normal price plots an exponential fit to the actual price. It is the price around which the actual price is expected to oscillate. It is shown here as the red line. The graph is in logs and the normal price is estimated over a period of three or more years. The normal price estimate is quite stable.



2. **Deviations from the normal price**
   The distance from the actual price to the normal price is the mispricing. It is a robust estimation.



3. **Crash/rally size**
   We assume that crashes or rallies are jumps in asset prices that are separate from typical random walk price movements. The crash/rally size is the average fraction of the distance from the normal price that the asset price is expected to jump back to. It is estimated on historical data and can depend on the size of the mispricing.



4. **Crash/rally probability**
   Two return parameters are needed for estimating the crash/rally probability: 1) the expected return if there is no crash and 2) the expected return after the crash. The probabilities are most sensitive to the first and less so to the second. The bubble model equates these expected returns with a function of the probability times the mispricing. This is called the Rational Expectations Condition (RE). The crash probability as a function of the mispricing can be calculated historically but it can also be calculated through the RE equation such that an increasing expected return causes an increasing crash probability. This means that bubbles that were never seen historically can properly be estimated.

$$p_{t+1} = p_t \exp(\bar{a}_t + \sigma\varepsilon_t) \quad \text{with} \quad p_0 > 0$$

and

$$\bar{a}_t = \begin{cases} \bar{r}_t & \text{with probability } 1-\rho_t \quad \text{with} \quad 0 \le \rho_t < 1 \\ \kappa_i \ln(q_t) + r_D & \text{with probability } \rho_t \eta_i \quad i = 1, 2, \ldots, n \\ & \kappa_i \in \Omega \equiv \{\kappa_i \mid -\infty < \kappa_i < \infty,\ i = 1, 2, \ldots, n\} \end{cases}$$

with

$$q_t = \frac{N_t}{p_t} \quad \text{and} \quad \sum_{i=1}^{n} \eta_i = 1 \quad 0 < \eta_i < 1 \text{ and } \bar{K} = \sum_{i=1}^{n} \eta_i \kappa_i$$

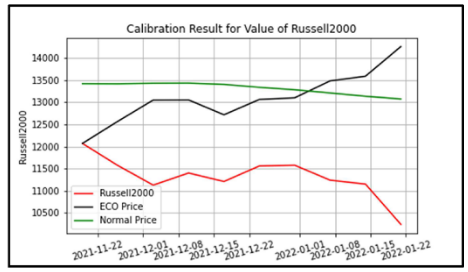
$$N_t = p_0 \exp(r_N t)$$

5. **Kelly Criterion and asset hedging**
   The Kelly Criterion has been used by gamblers and sophisticated asset allocators for over 60 years. When applying Kelly to a gambling situation with knowledge of the odds (probability) and edge (size) exactly, we know it will eventually outperform all other methods. Basically, the method simply means bet more when the odds are in your favor and less when they are not. In finance, we simply maximize the expected log of the future outcome of wealth when

allocated between the risky asset and cash. When the probability of a crash or the size of the mispricing increases, the output will suggest reducing exposure to the asset.

6. **Model calibration**

The big problem is that we cannot calculate the odds (probability) and edge (size) of an asset price exactly. We know that price returns are very difficult to estimate correctly. To generate these estimates, we use AI to sweep through roughly 3 or more months of asset price movements using 4 or 5 parameters in many different combinations to calculate expected returns. We pick the set of parameters that produce the best outcome. We look for almost stationary regimes. We use those parameters values to project what will happen in the next period. The graph provides an example of the outperformance versus the price using the calibration process.



7. **Performance**

Further discussion of the process as well as the success for timing exposure to various assets can be found in several peer reviewed technical papers. See https://riskontroller.com/info-center/). Current examples with weekly forward-looking projections are given at https://riskontroller.com/performance/ . The use of ECO in the AugurMax pro Forma portfolio is given at https://riskontroller.com/pro-forma-returns/ .

8. **Is it trend following?**

ECO is not a trend following strategy, in general. Of course, when prices keep increasing it may naturally appear like one. However, the similarity ends when the mispricing increases, or when the crash probability is rising and the allocation to the risky asset throttles back.

9. **Can model recommendations swing wildly?**

It depends on the volatility of the asset price. Highly volatile prices may require more frequent ***ECO*** runs. In other situations, 2 to 3 weeks can be optimal. The example in the Bitcoin paper shows the recommendations gradually decreasing toward a zero allocation prior to the 2018 crash. On the other hand in current estimations, Bitcoin has substantial price volatility and allocations swing more frequently.

10. **Model limitations**

The model is probabilistic, so it does not hedge bubbles correctly all the time. Furthermore, the process tends to estimate crashes better than rallies. When used over time, the ***ECO*** runs tend to outperform.

Comments, criticisms, questions to info@RisKontroller.com
Papers at https://riskontroller.com/info-center/

ECO Explanatory Notes and some Details

ECO Forecast Outputs Report Explanations

Item	**Description**	**Details and Use**
OutPerf %	*The percent by which the ECO trial run with the hedge value beats the asset price.*	The ECO trial run uses calibrated parameter values and should show an OutPerf% > 0. A value < 0 indicates a poor estimation with less reliability. See **Using ECO Scores**.
Hedge	*A value > 1 leverages the asset. A value < 0 shorts the asset.*	The optimal hedge obtained by the Kelly Criterion is the weight of the asset relative to the risk-free asset. The risk-free asset can be cash with zero return.
ECO % Correct	*ECO parameters are calibrated over an historical training set to determine optimal forecasts.*	The percentage of correct times the optimally trained set of parameters gives a positive performance on the set of determined historical intervals. We target >60% for good performance.
Correction Size	*A positive value means a crash could come and a negative value implies a rally could come.*	This is the fraction of the distance from the asset price to the normal price the asset price is expected to correct to. See **Using ECO Scores.**
Correction Prob%	*The probability that the specified correction size will occur.*	The probability that a correction can occur. This can be useful in constructing risk profiles.
OutPerf UB %	*The percent by which the ECO unbounded hedge beats the asset price over the trial period.*	In classical Kelly, the unbounded hedge gives the best limiting outperformance, but it can be very volatile in the short term.
Hedge UB	*The Hedge unbounded. We use values between -10 to 11.*	The larger the absolute value the stronger is the signal to short or go long.
Distance to Normal%	*Percentage that the asset price deviates from normal price.*	The normal price is the price that the asset is expected to oscillate around. When multiplied by the Correction Size, it gives the percentage correction expected.
Return % No Crash	*The expected return on the asset when there is no correction.*	This is the expected return over the number of days forecast and can be used to construct uncertainty scenarios.
Expected Return%	*The expected return of the asset price over the days forecast.*	The expected return considering the correction and no correction scenarios.
Normal %/yr	*The return on the normal price.*	Expected annualized return of the normal price, which is the medium to long term return (annualized) of the asset.
Bubble Begin	*The date at which the bubble begins.*	The date when the asset price exceeds the normal price and stays there for a crash, or the normal price exceeds the asset price and stays there for a rally.
Bubble Type	*Possible crash, rally, or neutral.*	Crash = asset price > normal price; Rally = normal price > asset price; Neutral = asset price close to normal price.
What to Do	*Buy Fast, Buy, Hold, Sell, or Sell Fast.*	The call depends on the size of the Hedge UB and the correction size.
Days Forecast	*The number of days rebalancing occurs in the trial period.*	If the number is small, there is a lot of changes in the volatility and direction of the asset. See **Using ECO Scores**.
Trial Start Date	*The date the trail run begins. Default is 3 months.*	The trail is the period that the ECO hedge is run and compared with the asset price (See OutPerf %).
Date Forecast	*The date the forecast is made.*	This can be "todays" date or a historical date.

A sample output of an ECO run is given in the following table. This is the detailed output with summaries of these tables posted on our website. Refer to the section below on **Using ECO Scores** for more details on each line item in the table.

Index	OutPerf %	Hedge	ECO % Correct	Correction Size	Correction Prob%	OutPerf UB %	Hedge UB	Dist to Normal%	Return% No Crash	Expected Return%	Normal %/yr	Bubble Begin	Bubble Type	What to do	Days Forecast	Trial Start Date	Date Forecast
Gold	8.14	(1.00)	54.76	0.05	2.00	30.08	(10.00)	13.58	(0.31)	(0.30)	7.57	2022-06-02	Rally	Sell Fast	2	2022-07-25	2022-11-18
JETS	22.30	2.00	50.00	0.00	0.00	13.18	7.53	(1.05)	2.51	(1.47)	(11.08)	2022-11-18	Neutral	Likely Sell	6	2022-07-25	2022-11-18
Oil	28.63	(1.00)	45.24	0.26	2.00	18.16	(9.96)	(9.88)	(1.52)	(1.57)	9.84	2022-06-02	Crash	Sell Fast	2	2022-07-25	2022-11-18
R1G	22.12	2.00	75.00	0.05	2.00	26.91	4.00	29.23	0.95	0.98	18.52	2022-06-02	Rally	Buy Fast	10	2022-07-29	2022-11-18
R2V	31.33	2.00	87.50	0.05	2.00	58.09	7.00	10.67	1.64	1.65	12.36	2022-08-18	Rally	Buy Fast	10	2022-07-29	2022-11-18
RWR	37.18	2.00	70.00	0.05	2.00	106.66	2.56	13.19	0.29	0.30	3.16	2022-08-25	Rally	Buy	8	2022-07-29	2022-11-18
TSLA	69.02	(1.00)	75.00	0.05	2.00	143.12	(3.80)	130.88	(5.97)	(5.83)	94.82	2022-06-02	Rally	Sell Fast	10	2022-07-29	2022-11-18
USInvBnd	21.85	2.00	78.57	0.05	2.00	113.99	11.00	15.12	0.87	0.88	1.61	2022-06-02	Rally	Buy Fast	6	2022-07-25	2022-11-18
XLK	23.11	2.00	62.50	0.05	3.00	42.41	5.78	29.96	1.71	1.75	22.33	2022-06-02	Rally	Buy Fast	10	2022-07-29	2022-11-18
XLV	9.91	1.73	57.14	0.05	2.00	(3.54)	1.73	6.02	0.02	0.03	12.56	2022-06-02	Rally	Buy	2	2022-07-25	2022-11-18
BTC	18.11	0.74	70.00	0.05	7.14	18.11	0.74	125.57	(0.16)	0.33	59.71	2022-06-02	Rally	Hold	8	2022-07-29	2022-11-18

Background

The last financial crisis (2007-2009) revealed serious flaws in economic modelling and in the use of mathematical and engineering models in finance, with respect to the occurrence of bubbles, crashes, and rallies. Here we apply our new bubble and crash model, which can be calibrated and made operational in portfolio investments. The model stresses the importance of positive feedback, the tendency for financial markets to self-correct only at long time scales while exhibiting significant departure from "normality" at short times.

In academia, discussions on financial bubbles often start with a reference to the Efficient Market Hypothesis (EMH), which in essence states that prices of financial assets properly reflect underlying economic fundamentals. Financial bubbles and the crashes that frequently follow them are arguably the most vivid challenge to the EMH. Our bubble model is a rational expectation model giving a relationship between the price process and a possible crash or a rally.

Our bubble model has the following important features:

1. It is a Rational Expectations model.
2. Prices temporarily deviate from a fundamental value or a "normal price" process.
3. It is mildly explosive using historical crash/rally probabilities.
4. It can become super-exponential, following an accelerating price path that would end in a crash when probabilities are computed dynamically via a feedback process.
5. The price stochastically oscillates around a normal price until it randomly begins to grow and then may accelerate to a bubble (positive or negative) or slowly correct back to the normal price.
6. It is dynamic as the stochastic properties change with the development of the price process.

It also has the following secondary features:

1. The price growth converges in the limit to that of the normal price process.
2. It combines a geometric random walk with a discrete Poisson distribution of crashes/rallies.
3. The crash/rally distribution sizes allow for over- and under-shooting the normal price.
4. It shows how bubbles can be spontaneously initiated and terminated.
5. It can be tested empirically by implementing an optimal investment method, which demonstrates a superior bubble mitigation performance in our OutPerf% metric.
6. It is arbitrage free.

We estimate the normal price in the sense of Merton (1980) by fitting an exponential return over an historical period of the asset price where the price stochastics have remained relatively constant. That historical period may end earlier than "today's" date with the normal price extrapolated up to today.

Our bubble model suggests that investment in the bubble is rational given the expectation that players can sell off at a higher price in the future before the bubble bursts. Yet, some players may get out as the crash probability increases beyond their risk threshold resulting in a plateau of prices before bursting.

We do not worry about defining exactly what a bubble is in terms of size. Instead, we separate the price asset history into Brownian Motion plus jumps. We therefore calculate the historical jump size relative to the normal price along with the jump probability (Jacquier and Okou. 2013) and focus on jumps.

We are not concerned with getting out of a perceived bubble. Rather we are interested in what to do when we are in a bubble. To do that we apply the Kelly Criterion (Kelly, 1956; MacLean, L. et al., 2010), which simply states that bet more when the odds are in your favor and less when they are not. We calculate those odds dynamically in terms of jump size and probability.

Kelly strategies in continuous time with jump-diffusion processes have been studied by Davis and Lleo 2013 and 2015). The fractional Kelly strategy puts a fraction of the wealth into the strategy, which makes the results less volatile but may not be optimal in the long run. Instead, we use a limit on the hedge size to control volatility.

What we do different from Davis and Lleo (2013 and 2015) is that we combine our bubble model with a discrete time Kelly process with dynamic variation in the jump distribution. Their jumps are independent of a bubble model and their model is in continuous time.

Papers on the methodology can be found in the references and additional materials can be found at https://riskontroller.com/info-center/ . The main reference on the method discussed here is found in Kreuser and Sornette, 2017.

The Model

We introduce the simple stochastic price process with a discrete Poisson process.

$$
\begin{gathered}
p_{t+1} = p_t \exp\left(\bar{a}_t + \sigma\varepsilon_t\right) \quad \text{with} \quad p_0 > 0 \\
\text{and} \\
\bar{a}_t = \begin{cases} \bar{r}_t & \text{with probability } 1-\rho_t \quad \text{with } \ 0 \le \rho_t < 1 \\ \kappa_i \ln\left(q_t\right) + r_D & \text{with probability } \rho_t\eta_i \quad i = 1,2,\ldots,n \\ & \kappa_i \in \Omega \equiv \left\{\kappa_i \middle| -\infty < \kappa_i < \infty,\ i = 1,2,\ldots,n\right\} \end{cases} \\
\text{with} \\
q_t = \frac{N_t}{p_t} \quad \text{and} \quad \sum_{i=1}^{n} \eta_i = 1 \quad 0 < \eta_i < 1 \text{ and } \bar{K} = \sum_{i=1}^{n} \eta_i \kappa_i \\
N_t = p_0 \exp\left(r_N t\right)
\end{gathered} \tag{1}
$$

The crash factors are assumed independent and dynamic changing over time and distributed according to the probability distribution $\Pi \equiv \{\eta_i = \Pr[\text{crash amplitude} = \kappa_i] \| \ i = 1, 2, ..., n\}$. Thus, conditional on no crash happening, which holds at each time step with probability $1-\rho_t$, the price p_t follows a geometric random walk with mean return $\overline{r_t}$ on Δt and volatility σ. At each time step, there is a probability ρ_t for a crash/rally to happen with an amplitude that is proportional to the bubble size, defined as $\ln(q_t) = \ln\left(\frac{N_t}{p_t}\right)$ where $N_t = p_0 \exp(r_N t)$ and r_N is defined as the long-term average return normal[1] price.

The values $\bar{K}, \rho_t, N_t$ are determined over a rolling historical period with ρ_t further defined by price accelaration. The parameter Δt along with the historical period and the estimation of $\overline{r_t}$ and r_D are calibrated so that application of the Kelly Criterion over a short trial period outperforms thus assuming that the current paramter estimations are reasonably good.

The parameters $\overline{r_t}, r_D, \rho_t$ are subsequently adjusted to satisfy the Rational Expectation condition

$E_t\left(\ln\left(\frac{p_{t+1}}{p_t}\right)\right) = r_D \quad \forall t$, which reads

$$E_t\left[\ln\left(\frac{p_{t+1}}{p_t}\right)\right] = (1-\rho_t)\overline{r_t} + \rho_t\left[\left(\sum_{i=1}^{n} \eta_i \kappa_i\right)\ln(q_t) + r_D\right] = r_D \qquad (2)$$

The "Hedge" (also called lambda, λ_t) is computed to satisfy the Kelly Criterion

$$\max_{\lambda_t} E_t\left[\ln\left(\frac{W_{t+1}}{W_t}\right)\right]$$
$$\textit{with}$$
$$W_{t+1} = \left(\lambda_t \exp(\overline{a}_t + \sigma\varepsilon_t) + (1-\lambda_t)\exp(r_f)\right)W_t$$

where W_t is the wealth at time t and r_f is the risk-free rate or zero.

For the "Hedge" value we use $-1 \le \lambda_t \le 2$ and for the "Hedge UB" we use $-10 \le \lambda_t \le 11$. We recommend the use of the range [-1,2] in general because the result is less volatile.

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[1] We call it the "normal price return". Some may interpret this as a fundamental price return but that is not the specific intention here.

Using ECO Scores

We refer here to the table describing each element of the ECO Forecast Report. When we refer to the ECO Portfolio, we mean the portfolio consisting of a combination of λ_t of the asset plus $(1-\lambda_t)$ of the risk-free asset where the value λ_t is the Hedge value.

1. First check the column 'OutPerf %' as it should be positive. A set of three parameters are automatically calibrated and the ECO method is run over the trial period (usually about three months). The value of OutPerf% shows the percentage outperformance of the method over the trial period. A positive value means that he parameter selection has likely been optimally chosen. A negative value means that the parameters did not calibrate well and the result in the "What to Do" column is to "Hold Fast" meaning to keep the current allocation. Further information on parameter calibration will be available in a separate document.
2. The "ECO % Correct" indicates the probability that the recommended hedge on each of an historical set of intervals will outperform over the intervals (the number of days in an interval is given by the "Days Forecast".). The historical set of intervals is determined by an AI algorithm to give the best outcome over that period. See **Model Calibration** on page 2 and 11 below.
3. Refer to the column 'What to do'. The directions Fast or Slow depend on the relative size of 'Lambda UB' and other parameters and indicate buy, sell, or hold and their speed of action.
4. The column 'Expected Return%' is the value for r_D and the 'Return% No Crash' is the value of $\overline{r_t}$. Both returns are taken over the interval Δt given by the value in the column 'Days Forecast'. The value of Δt refers to number of days between rebalances used in the trail period. It also is an estimate of the number of days until a rebalance is needed in the future. A small value indicates that the price path is stochastically changing rapidly and requires frequant rebalancing.
5. Confirm the following indicators:
   a. Do the hedge and the expected return values have the same sign? If they do not, then the model output does not warrant a buy or sell and the action should be neutral or 'hold.' This is also used in the determination of the 'What to Do'.
6. How much risk is there?
   a. Ascertained in part from the mispricing, which is the 'Dist to Normal%' in percent terms $= 100\ln\left(\frac{N_t}{p_t}\right)$. The expected correction size is given by the 'Correction Size' ($\overline{\mathrm{K}}$) times the 'Dist to Normal%'.
7. How likely is the bubble to burst?
   a. The 'Correction Prob%' = $100\rho_t$ gives the probability of the bubble correcting.
8. The 'OutPerf UB %' and 'Hedge UB' give the corresponding values when the hedge is allowed to vary between -10 to +11. This is a more risky approach but may lead to outsized returns. The examples in the table did well in this case. However, we do not recommend using those values in general because they can give very volatile results.
9. What happens if the bubble does not burst?
   a. We have an expected return given by 'Return% No Crash'. A risk manager can therefore assess how much risk to take in buying or selling an asset using this value.
   b. Of course expectations will not correspond exactly with actual returns. Ranking assets with higher expected returns however typically corresponds with actual higher returns.

10. The ‘Normal %/yr’ is the normal price return annualized. One may check how close that is to perceived annualized expected return of the asset.
11. Other returns can be annualized by multiplying by 252 divided by the ‘Days Forecast’.
12. The ‘Bubble Type’ is determined by the sign of the ‘Dist to Normal%’.
13. The ‘Bubble Begin’ is the date that the asset price exceeds by a small tolerance the normal price for a crash state or the normal price exceeds the asset price for a rally state.
14. The ‘Trial Start Date’ is the date at which the trial to determine outperformance for the calibrated parameters begins.
15. The ‘Date Forecast’ is the date that the forecast is made. It may not necessarily be todays date as it is possible to make historical runs.
16. The values will satisfy the Rational Expectation condition taken directly from the table as follows:

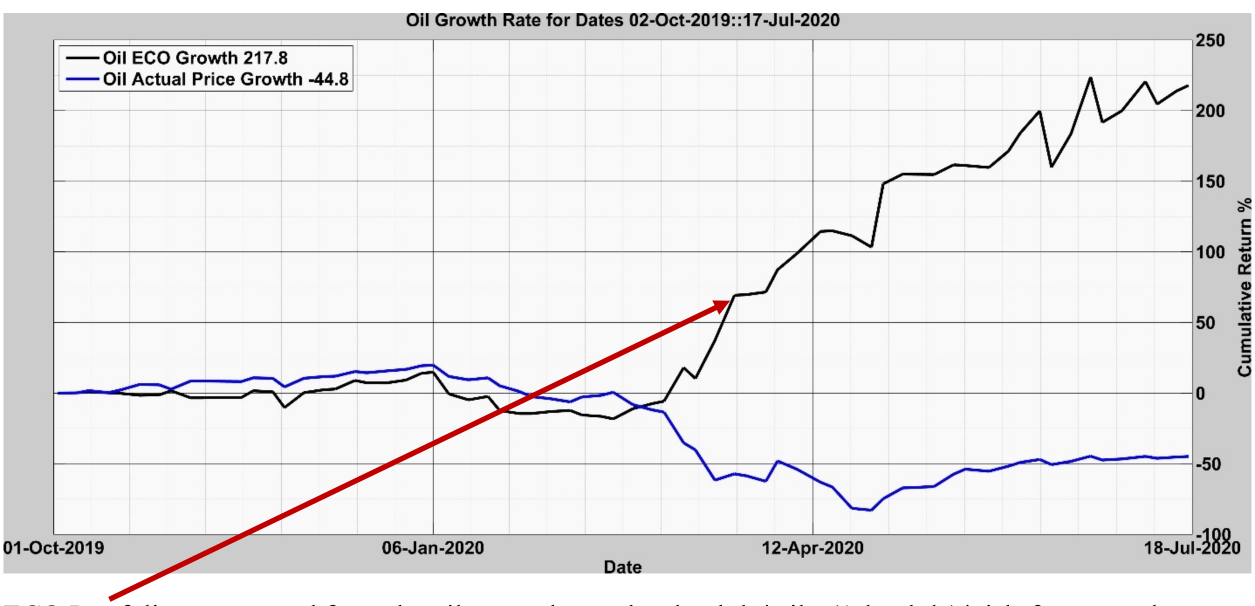
$$\bar{r}_t = r_D - \left(\frac{\rho_t}{100-\rho_t}\right)\bar{K}\ln\left(\frac{N_t}{p_t}\right)$$

This calculation holds on all the assets as you can see in the Sample Output Table.

ECO Graphical Examples

The ECO process entails calibrating a parsimonious set of model parameters to obtain results with a better likelihood of estimating the bubble statistics. The model is run for a short time (around 3 to 4 months) to calibrate it. These parameters are expected to hold for some months as the price process goes through a rather stable regime. A regime is stable if the price stochastics do not change considerably. ‘Stable’ here does not imply that the price process goes though time without volatility.

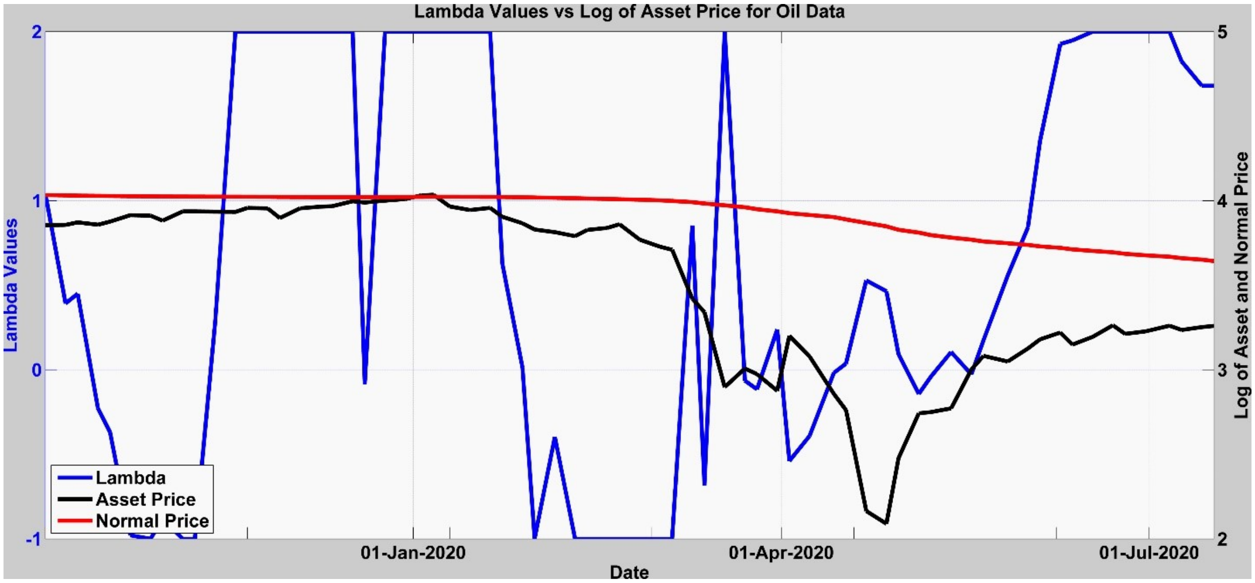
Using ECO output, we see substantial outperformance for timing exposure to West Texas Intermediate (WTI) oil from October 2019 through July 2020 in the chart below. We use ECO values as given in the ECO Forecast Report to project what will happen in the future (Bubble crash, rally, returns, etc. as in the table).



ECO Portfolio constructed from the oil asset plus cash = lambda*oil +(1-lambda)*risk-free or cash.

We are using a zero return on cash. Lambda is the hedge value.

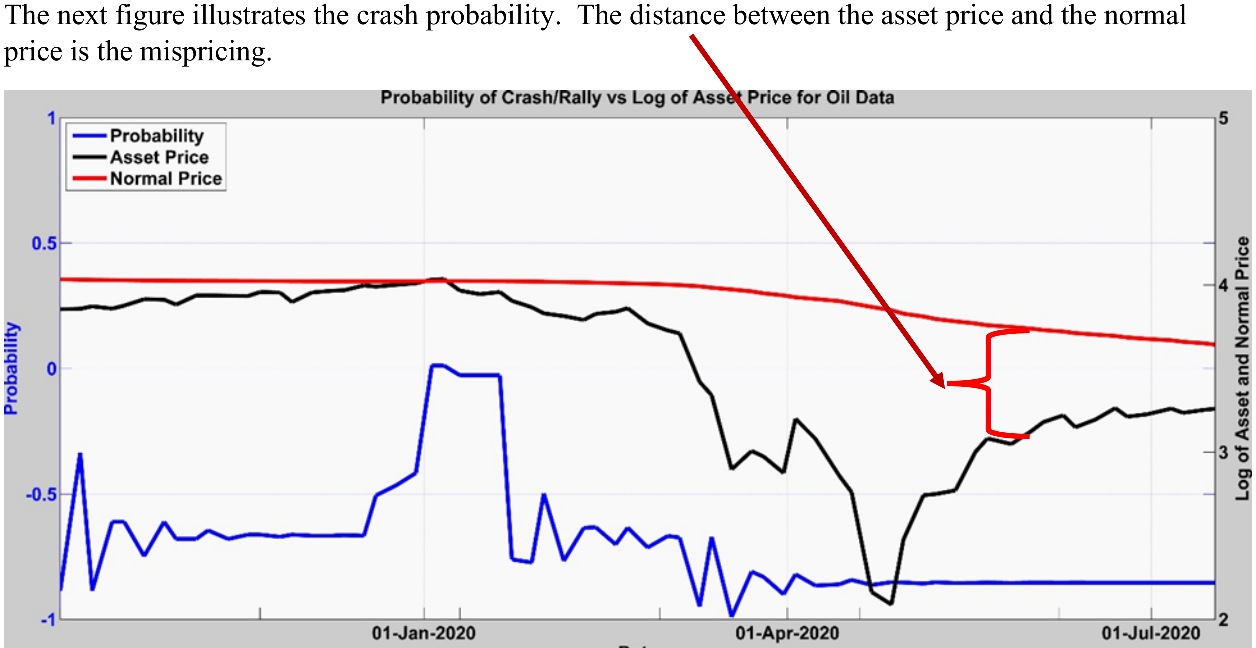
Lambda values are graphed in the next figure. A value of -1 means the asset price is likely to decline and that one should be invested in cash and not in oil. A value of -1 also means that it is a good time to sell oil or short it.



Correction probabilities are given in the next figure. A crash is a positive probability. A rally probability is recorded as negative to distinguish it from a crash probability.

Note that a rally probability near 1 (illustrated as a negative) can coincide with a lambda near 2. This can occur because a rally has occurred and is expected to continue because the asset price is increasing. Since it is increasing, it can be worthwhile to leverage that asset, i.e., λ_t has a value of 2 or > 1.

The next figure illustrates the crash probability. The distance between the asset price and the normal price is the mispricing.



You can see when the crash probability decreases toward -1 (where rally probabilities are illustrated with a minus sign to distinguish them from crash probabilities) in relation to a declining asset price.

The bubble size is the expected percentage jump back towards the normal price.

The value of the probability and lambda are obtained assuming the rational expectation conditon holds:

$$E_t\left[\ln\left(\frac{p_{t+1}}{p_t}\right)\right]=\left(1-\bar{\rho}\right)\bar{r}_t+\bar{\rho}\bar{K}\ln\left(\frac{N_t}{p_t}\right)+\bar{\rho}r_D=r_D$$

The lambda value is obtained by maximizing the expected growth rate of wealth assuming that the portfolio consists of the asset and cash. The result gives the optimal combination of the asset and cash but it also gives the indicators on whether or not the asset should be bought, sold, or just held constant and by how much, that is **What to Do When You are in a Bubble!**

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