TIME IS THE ESSENCE

Ramasastry Ambarish and Lester Seigel **explain that fund managers' performance claims may depend critically on the time period examined**

To assess the performance of a portfolio or contract against a benchmark requires first selecting the most appropriate period for the comparison. We call this period the "report card time" and we consider it here in the context of the Australian Stock Exchange's (ASX's) share ratio contract[1].

The price of this contract is the ratio of the individual share price to the value of the All Ordinaries Index, multiplied by 1,000. The contract serves to capture any outperformance (or underperformance) of that share against the market. But how long would it take to judge whether a strong performance in an ASX contract was the result of skill (signal) or just luck (noise)? The issue clearly goes beyond the specifics of the ASX contracts, applying equally to managed portfolios' performance against benchmarks.

The latter can be seen as a race between two noisy growth processes – the portfolio and the index against which it is compared – which are fully intertwined as they compete. In the short run, even if the deterministic (ie, skilled) component of the managed portfolio's performance exceeds that of the index, the chances are high that the success story will be buried in the random fluctuations of both portfolio and index.

Over time, however, the signal will emerge over the noise and the clever portfolio selection will be recognised. The question is, how long will this take – months, years or even centuries – and what are the key determinants of this timescale? Before addressing this central theme, we will briefly discuss the merits of a ratio measure of relative performance.

It is traditional in finance to treat the risk of a financial instrument as the variance of its rate of return, thus capturing the squared standard deviation from the instrument's mean rate of return. Therefore, when the risk is *relative*, a logical extension of this thinking would be to define it as the variance per unit time, σ_R^2, of the difference between the rates of return of the managed portfolio and of the index. Labelling the portfolio and index values as P and I, respectively, this suggested measure of relative risk is:

$$\sigma_R^2 = \text{variance per unit time of } \left(\tfrac{dP}{P} - \tfrac{dI}{I}\right) \quad (1)$$

On this basis, the ratio method of valuing relative risk takes centre stage. For, if the portfolio:index ratio is itself treated as a financial instrument, as in the ASX share ratio contracts, then the variance of the rate of return of such ratios will be identical to the measure of relative risk in equation (1). It is this feature that makes the ratio measure so special. Some maths will make this clearer, while setting the framework for the time horizon discussion.

Let the portfolio (P) and index (I) both follow the usual generalised Wiener process, so that:

$$\frac{dP}{P} = \mu_P dt + \sigma_P dz_P \qquad (2)$$
$$\frac{dI}{I} = \mu_I dt + \sigma_I dz_I$$

where (μ_P, σ_P) and (μ_I, σ_I) are instantaneous mean and volatility parameters of the portfolio and index, respectively. If $\rho_{I,P}$ is the coefficient of correlation between the portfolio and the index, the dynamics of the ratio measure, $R(t) = \frac{P(t)}{I(t)}$, can be uncovered through the application of Itô's lemma, to find:

$$\frac{dR}{R} = (\mu_P - \mu_I + \sigma_I^2 - \sigma_I\sigma_P\rho_{I,P})\,dt + \sigma_P dz_P - \sigma_I dz_I \quad (3)$$

From the above, it is clear that the variance of the stochastic terms in equation (3) is identical to equation (1), completing the intuitive appeal of using R(t) as the measure of relative performance.[2]

Turning to the time development of R(t), it is convenient to define the stochastic variable, dz:

$$\sigma_R dz \equiv \sigma_P dz_P - \sigma_I dz_I,$$

where: (4)

$$\sigma_R^2 = \sigma_I^2 + \sigma_P^2 - 2\sigma_I\sigma_P\rho_{I,P}$$

Then, as in simple geometric Brownian motion for stocks, the solution of equation (3) is:

$$R(t) = R(0)\exp\left[\sigma_R\varepsilon\sqrt{t}\right]\cdot \exp\left[\left\{\left(\mu_P - \frac{\sigma_P^2}{2}\right) - \left(\mu_I - \frac{\sigma_I^2}{2}\right)\right\}t\right] \quad (5)$$

where ε is the standard normal variable. The first exponential factor in equation (5) is the noise and the second is the signal. From equation (5), note that for sufficiently small t, the noise predominates and asymptotically the signal prevails.

Before we continue our analysis, we need to examine the claims made by some fund managers that they consistently outperform the market. In particular, they boast that their superior market timing, instrument selection, information processing and so on mean their rate of return (which we have labelled μ_P) is sufficiently in excess of μ_I that their enhanced performance will be seen over reasonable time horizons (ie, quarterly, six-monthly or annually). Let's review the numerical implications of this claim by gaining a sense of the length of time needed for signal to exceed noise.

The criterion for the signal to dominate the noise is that the time horizon (report card time), T, should be large enough for the signal exponent to exceed that of the noise, or equivalently[3]:

$$T > \frac{K^2\left(\sigma_I^2 + \sigma_P^2 - 2\sigma_I\sigma_P\rho_{I,P}\right)}{\left[\left(\mu_P - \frac{\sigma_P^2}{2}\right) - \left(\mu_I - \frac{\sigma_I^2}{2}\right)\right]^2} \qquad (6)$$

where K is the number of standard deviations for a given confidence level.

Some numbers may be helpful at this point in illustrating the manager's dilemma. Let K = 1 (corresponding to a confidence level of 84%). Select $\sigma_I = 0.15$, $\sigma_P = 0.25$ and $\rho_{I,P} = 0.9$, and further assume that the rate of return of the portfolio outperforms the benchmark by a robust 300 basis points, ie, $\mu_P - \mu_I = 0.03$. Feeding these values into equation (6) reveals that T > 175 years! In other words, in about 175 years, the relative performance signal of the manager would exceed the noise about 84% of the time. In contrast, a comparable calculation over one year reveals that the noise fully dominates the signal, rendering highly questionable any judgement on such short-term performance.

Many other claims by fund managers are revealed as myths by equations (4) and (6). As a second illustration, managers often seek to reduce portfolio volatility (σ_P) and concurrently increase the rate of return μ_P. However, the focus of the present exercise is on relative risk and hence it is σ_R^2 that needs to be

[1] The structure of the contract is described by Jane Locke in "Relative Values" (*Risk* August 1995, page 21) and its valuation by David Elms in "Rationale of Ratios" (ibid, page 24)

[2] The lack of apparent symmetry in the expression for the rate of return of R(t) in equation (3) is due to the fact that R(t) is non-linear in I(t) but linear in P(t). The symmetry can be restored by focusing on the difference in asymptotic growth rates. Namely:

$$\mu_R = \left(\mu_P - \frac{\sigma_P^2}{2}\right) - \left(\mu_I - \frac{\sigma_I^2}{2}\right) + \frac{\sigma_R^2}{2}$$

[3] The discussion assumes the performance of the fund manager can be characterised as a geometric Brownian motion (equation 2); however, the central theme holds for a wide class of stochastic models

reduced. From equation (4), the minimum value of σ_R^2 occurs when $\sigma_P = \sigma_I\rho_{I,P}$. This correlation yields a dramatic reduction in the report card time to 4.1 years, although this is still substantial. Thus, reducing portfolio volatility without considering correlation may prove counterproductive.

The above arguments are not meant to be an "impossibility theorem", advocating the abandonment of performance measurements. But the message is clear: when assessing performance relative to a benchmark over modest time windows, proceed with caution. The lengthy report card time implies that any attempt to assess relative performance over short horizons is fraught with difficulties. Ironically, it is precisely the relatively high level of noise that underlies this lengthy report card time that makes the ASX share ratio contracts attractive trading instruments.

One can quickly derive the fair value of the ASX contracts by taking the expectation of R(t) in the risk-neutral world, using the identity $E[\exp(\sigma_R \varepsilon \sqrt{t})] = \exp(\sigma_R^2 t/2)$. Then if g is the scaling factor, the fair value of an ASX share ratio contract[4,5] expiring at time t is given by:

$$g\frac{P_f}{I_f}\exp(\sigma_I^2 - \rho_{I,P}\sigma_I\sigma_P)t \qquad (7)$$

where P_f and I_f are the forward prices of the share and index respectively. Readers familiar with the valuation of "quanto forward" contracts will immediately recognise the convexity factor – this typically appears in the valuation of contingent claims whose payout is normalised by the value of another security.[6]

We have shown that the ASX contracts provide a model for comparing portfolio/index performance. We have also pointed out that unless the benchmark/index correlation and rate-of-return spread are extremely large, report card times can be substantial. Finally, with all the attention now focused on value-at-risk and risk measurement units, the above methodology can easily be extended to relative risk measurement units. ■

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[4] This is equation (5) in David Elm's article (see footnote 1)
[5] In this context, P is the individual share and I is the All Ordinaries Index
[6] For instance, see Paul Doust's "Relative Pricing Techniques in the Swaps and Options Markets", *The Journal of Financial Engineering*, Vol 4 (1), pages 11–46